Integration Bee

Round of 16

January 2026

Question 1

$$\int \sin(2026x)\sin^{2024} x \, dx$$

Proposed by **Rahul Ganguly** *(BStat 3rd year)*

$$\int \sin(2026x) \sin^{2024} x \, dx$$

$$= \int \sin(2025x + x) \sin^{2024} x \, dx$$

$$= \int \left(\sin(2025x) \cos x + \cos(2025x) \sin x\right) \sin^{2024} x \, dx$$

$$= \int \sin(2025x) \sin^{2024} x \cos x \, dx + \int \cos(2025x) \sin^{2025} x \, dx$$

$$= I_1 + I_2$$

$$I_1 = \int \sin(2025x) \sin^{2024} x \cos x \, dx \qquad I_2 = \int \cos(2025x) \sin^{2025} x \, dx$$

Evaluation of I_1

$$I_1 = \int \sin(2025x) \sin^{2024} x \cos x \, dx$$

$$u = \sin(2025x), \qquad dv = \sin^{2024} x \cos x \, dx$$

$$du = 2025 \cos(2025x) \, dx, \qquad v = \frac{\sin^{2025} x}{2025}$$

$$I_1 = uv - \int v \, du$$

$$\boxed{I_1 = \frac{\sin(2025x)\sin^{2025} x}{2025} - \int \cos(2025x)\sin^{2025} x\, dx}$$

$$I_1 + I_2 = \frac{\sin(2025x)\sin^{2025} x}{2025} - \int \cos(2025x)\sin^{2025} x\, dx + \int \cos(2025x)\sin^{2025} x\, dx$$

$$\boxed{\int \sin(2026x)\sin^{2024} x\, dx = \frac{\sin(2025x)\sin^{2025} x}{2025} + C}$$

—— End of Solution 1 ——

Question 2

$$\int_0^{2026} x\,|\sin(\pi x)|\,dx$$

Proposed by **Archisman Mukhopadhyay** *(MStat 2nd year)*

First we observe that, $|\sin(\pi x)|$ is periodic with period 1

Also, $$\int_0^{2026} x|\sin(\pi x)|dx = \sum_{k=0}^{2025} \int_k^{k+1} x|\sin(\pi x)|dx$$

On $[k, k+1]$, substitute,

$$x = k + t, \quad t \in [0,1]$$

$$\int_k^{k+1} x|\sin(\pi x)|dx = \int_0^1 (k+t)\sin(\pi t)\,dt$$

$$\int_0^1 (k+t)\sin(\pi t)\,dt = k\int_0^1 \sin(\pi t)\,dt + \int_0^1 t\sin(\pi t)\,dt$$

Introduce

$$I_1 = \int_0^1 \sin(\pi t)\,dt, \qquad I_2 = \int_0^1 t\sin(\pi t)\,dt$$

Evaluation of I_1

$$I_1 = \left[-\frac{\cos(\pi t)}{\pi}\right]_0^1 = \frac{-\cos\pi + \cos 0}{\pi} = \frac{2}{\pi}$$

Evaluation of I_2

$$I_2 = \int_0^1 t\sin(\pi t)\,dt$$

$$u = t, \quad dv = \sin(\pi t)\,dt, \quad du = dt, \quad v = -\frac{\cos(\pi t)}{\pi}$$

$$I_2 = \left[-\frac{t\cos(\pi t)}{\pi}\right]_0^1 + \frac{1}{\pi}\int_0^1 \cos(\pi t)\,dt$$

$$\Rightarrow I_2 = \frac{1}{\pi}\left[-\frac{t\cos(\pi t)}{\pi}\right]_0^1 + \frac{1}{\pi}\left[\frac{\sin(\pi t)}{\pi}\right]_0^1$$

$$\Rightarrow \boxed{I_2 = \frac{1}{\pi}}$$

So, $\displaystyle\int_k^{k+1} x|\sin(\pi x)|dx = k \cdot \frac{2}{\pi} + \frac{1}{\pi}$

$$\boxed{\Rightarrow \int_k^{k+1} x|\sin(\pi x)|dx = \frac{2k+1}{\pi}}$$

So, $\displaystyle\int_0^{2026} x|\sin(\pi x)|dx = \frac{1}{\pi}\sum_{k=0}^{2025}(2k+1)$

$$\sum_{k=0}^{2025}(2k+1) = 2\sum_{k=0}^{2025} k + 2026 = 2026^2$$

$$\boxed{\int_0^{2026} x\,|\sin(\pi x)|\,dx = \frac{2026^2}{\pi}}$$

—— End of Solution 2 ——

Question 3

$$\int_0^\infty \frac{\ln(1+x)}{x^{3/2}}\,dx$$

Proposed by Soham Gondane (Bstat 2nd year)

$$u = \ln(1+x) \quad \Rightarrow \quad du = \frac{dx}{1+x}$$

$$dv = x^{-3/2}dx \quad \Rightarrow \quad v = -2x^{-1/2}$$

$$\int_0^\infty \frac{\ln(1+x)}{x^{3/2}}dx = uv\Big|_0^\infty - \int_0^\infty v\,du$$

Boundary term:

$$uv\Big|_0^\infty = \Big[-2\frac{\ln(1+x)}{\sqrt{x}}\Big]_0^\infty = 0$$

Remaining integral:

$$-\int_0^\infty v\,du = 2\int_0^\infty \frac{dx}{\sqrt{x}(1+x)}$$

Let $x = t^2 \Rightarrow dx = 2t\,dt,\ \sqrt{x} = t$

$$2\int_0^\infty \frac{dx}{\sqrt{x}(1+x)} = 2\int_0^\infty \frac{2t\,dt}{t(1+t^2)} = 4\int_0^\infty \frac{dt}{1+t^2}$$

$$= 4\left[\arctan t\right]_0^\infty = 4\cdot\frac{\pi}{2} = 2\pi$$

$$\boxed{\int_0^\infty \frac{\ln(1+x)}{x^{3/2}}\,dx = 2\pi}$$

— End of Solution 3 —

Question 4

$$\int_0^1 \frac{1}{(1+x^2)^4}\, dx$$

Proposed by **Ayush Baran Sen** *(BStat 3rd year)*

$$\int_0^1 \frac{dx}{(1+x^2)^4}$$

Let

$$x = \tan t \quad \Rightarrow \quad dx = \sec^2 t\, dt, \qquad 1 + x^2 = \sec^2 t.$$

When $x = 0$, $t = 0$; when $x = 1$, $t = \frac{\pi}{4}$.
Hence,

$$\int_0^1 \frac{dx}{(1+x^2)^4} = \int_0^{\pi/4} \frac{\sec^2 t}{\sec^8 t}\, dt = \int_0^{\pi/4} \cos^6 t\, dt.$$

Using the identity

$$\cos^6 t = \frac{5}{16} + \frac{15}{32}\cos 2t + \frac{3}{16}\cos 4t + \frac{1}{32}\cos 6t,$$

we get

$$\int_0^{\pi/4} \cos^6 t \, dt = \int_0^{\pi/4} \left(\frac{5}{16} + \frac{15}{32} \cos 2t + \frac{3}{16} \cos 4t + \frac{1}{32} \cos 6t \right) dt.$$

Integrating termwise,

$$= \left[\frac{5}{16} t + \frac{15}{64} \sin 2t + \frac{3}{64} \sin 4t + \frac{1}{192} \sin 6t \right]_0^{\pi/4}.$$

Since

$$\sin\frac{\pi}{2} = 1, \quad \sin\pi = 0, \quad \sin\frac{3\pi}{2} = -1,$$

we obtain

$$\int_0^1 \frac{dx}{(1+x^2)^4} = \frac{5}{16}\cdot\frac{\pi}{4} + \frac{15}{64} - \frac{1}{192} = \frac{5\pi}{64} + \frac{11}{48}.$$

$$\boxed{\int_0^1 \frac{dx}{(1+x^2)^4} = \frac{5\pi}{64} + \frac{11}{48}}$$

—— End of Solution 4 ——

Question 5

$$\int \ln[(x-1)(x+3)(x-2)(x-6)+96]\,dx$$

Proposed by **Rahul Ganguly** *(BStat 3rd year)*

First observe that, $(x-1)(x-2) = x^2 - 3x + 2,$

$$(x+3)(x-6) = x^2 - 3x - 18$$

$$(x-1)(x+3)(x-2)(x-6) + 96 = (x^2 - 3x + 2)(x^2 - 3x - 18) + 96$$

$$\boxed{Let, a = x^2 - 3x}$$

So the expression becomes,

$$\begin{aligned} &(a+2)(a-18) + 96 \\ &= a^2 - 16a + 60 \\ &= (a-6)(a-10) \\ &= (x^2 - 3x - 6)(x^2 - 3x - 10) \end{aligned}$$

Now, $x^2 - 3x - 6 = \left(x - \dfrac{3+\sqrt{33}}{2}\right)\left(x - \dfrac{3-\sqrt{33}}{2}\right)$,

$x^2 - 3x - 10 = (x-5)(x+2)$

$$\begin{aligned}\ln[(x-1)(x+3)(x-2)(x-6)+96] = \ln\left(x - \tfrac{3+\sqrt{33}}{2}\right) + \ln\left(x - \tfrac{3-\sqrt{33}}{2}\right) \\ + \ln(x-5) + \ln(x+2)\end{aligned}$$

As, $\displaystyle\int \ln(x-a)\,dx = (x-a)\ln(x-a) - (x-a) + C$

$$\int \ln\big[(x-1)(x+3)(x-2)(x-6)+96\big]\,dx$$

$$= \left(x-\frac{3+\sqrt{33}}{2}\right)\ln\left(x-\frac{3+\sqrt{33}}{2}\right)$$

$$+\left(x-\frac{3-\sqrt{33}}{2}\right)\ln\left(x-\frac{3-\sqrt{33}}{2}\right)$$

$$+(x-5)\ln(x-5)+(x+2)\ln(x+2)-4x+C$$

—— End of Solution 5 ——

Question 6

$$\int_{-2016}^{2016} \frac{\dfrac{\sin(\pi x)}{e} + \dfrac{6x^7}{e} + \dfrac{8x^2}{\pi}}{x^6 + 1}\, dx$$

Proposed by **Ayush Baran Sen** *(BStat 3rd year)*

Split the integral into three parts

$$I = I_1 + I_2 + I_3$$

$$I_1 = \int_{-2016}^{2016} \frac{\sin(\pi x)}{e(x^6+1)}\,dx$$

$$I_2 = \int_{-2016}^{2016} \frac{6x^7}{e(x^6+1)}\,dx$$

$$I_3 = \int_{-2016}^{2016} \frac{8x^2}{\pi(x^6+1)}\,dx$$

Parity analysis

$\sin(\pi x)$ is odd, $\quad x^7$ is odd, $\quad x^2$ is even

$x^6 + 1$ is even

$$\Rightarrow I_1 = 0, \qquad I_2 = 0$$

Evaluate the remaining even integral

$$I_3 = 2\int_0^{2016} \frac{8x^2}{\pi(x^6+1)}\,dx$$

Substitute $t = x^3, \quad dt = 3x^2 dx$

$$I_3 = \frac{16}{3\pi}\int_0^{2016^3} \frac{dt}{t^2+1} = \frac{16}{3\pi}\tan^{-1}(2016^3)$$

$$\boxed{\int_{-2016}^{2016} \frac{\frac{\sin(\pi x)}{e} + \frac{6x^7}{e} + \frac{8x^2}{\pi}}{x^6+1}\,dx = \frac{16}{3\pi}\tan^{-1}(2016^3)}$$

—— End of Solution 6 ——

Question 7

$$\int_0^\infty \frac{x}{\sinh x}\, dx$$

Proposed by **Soham Gondane** *(BStat 2nd year)*

Use series expansion for $\dfrac{1}{\sinh x}$:

$$\frac{1}{\sinh x} = 2\sum_{n=0}^{\infty} e^{-(2n+1)x}, \qquad x > 0$$

So the integral becomes:

$$\int_0^{\infty} \frac{x}{\sinh x}\, dx = 2\sum_{n=0}^{\infty} \int_0^{\infty} x e^{-(2n+1)x}\, dx$$

Use the formula $\int_0^\infty xe^{-ax}dx = \frac{1}{a^2},\ a > 0$

$$\Rightarrow \int_0^\infty xe^{-(2n+1)x}dx = \frac{1}{(2n+1)^2}$$

Hence the series becomes:

$$\int_0^\infty \frac{x}{\sinh x}\,dx = 2\sum_{n=0}^{\infty} \frac{1}{(2n+1)^2}$$

Use the known series $\displaystyle\sum_{n=0}^{\infty} \frac{1}{(2n+1)^2} = \frac{\pi^2}{8}$

$$\Rightarrow 2\sum_{n=0}^{\infty} \frac{1}{(2n+1)^2} = 2 \cdot \frac{\pi^2}{8} = \frac{\pi^2}{4}$$

Therefore, the integral evaluates to:

$$\boxed{\int_0^{\infty} \frac{x}{\sinh x}\, dx = \frac{\pi^2}{4}}$$

—— End of Solution 7 ——

Question 8

$$\int e^{x^2} x \sin x^2 \, dx + \int e^{x^2} \pi x \cos x^2$$

Proposed by **Ayush Baran Sen** *(BStat 3rd year)*

$$\int e^{x^2} x \sin x^2 \, dx + \int e^{x^2} \pi x \cos x^2 \, dx$$

$$= \int e^{x^2} x \left(\sin x^2 + \pi \cos x^2\right) dx$$

Let

$$x^2 = t \quad \Rightarrow \quad dt = 2x \, dx \quad \Rightarrow \quad x \, dx = \frac{dt}{2}$$

$$I = \frac{1}{2} \int e^t \left(\sin t + \pi \cos t\right) dt$$

$$I = \frac{1}{2} \left(I_1 + \pi I_2\right)$$

$$I_1 = \int e^t \sin t \, dt$$

Integrating by parts,

$$I_1 = e^t(\sin t - \cos t) - I_1$$

$$\Rightarrow \quad 2I_1 = e^t(\sin t - \cos t)$$

$$\boxed{I_1 = \frac{1}{2}e^t(\sin t - \cos t)}$$

Similarly,

$$I_2 = \int e^t \cos t \, dt$$

$$\boxed{I_2 = \frac{1}{2}e^t(\cos t + \sin t)}$$

$$I = \frac{1}{2}\left[\frac{1}{2}e^t(\sin t - \cos t) + \pi \cdot \frac{1}{2}e^t(\cos t + \sin t)\right]$$

$$= \frac{1}{4}e^t\Big((1+\pi)\sin t + (\pi - 1)\cos t\Big)$$

$$\boxed{I = \frac{1}{4}e^{x^2}\left[(1+\pi)\sin x^2 + (\pi - 1)\cos x^2\right] + C}$$

—— End of Solution 8 ——

Integration Bee

Round of 8

January 2026

Question 1

$$\int_0^\infty \frac{\sin^4 x}{x^2}\, dx$$

Proposed by **Ayush Baran Sen(Bstat 3rd year)**

$$\int_0^\infty \frac{\sin^4 x}{x^2}\,dx = \left[-\frac{\sin^4 x}{x}\right]_0^\infty + \int_0^\infty \frac{4\sin^3 x\cos x}{x}\,dx$$

(Integration by parts: $u = \sin^4 x,\ dv = \dfrac{dx}{x^2}$)

$$\left[-\frac{\sin^4 x}{x}\right]_0^\infty = 0$$

$$\Rightarrow \int_0^\infty \frac{\sin^4 x}{x^2}\,dx = \int_0^\infty \frac{4\sin^3 x\cos x}{x}\,dx$$

$$\sin^3 x = \frac{3\sin x - \sin 3x}{4}$$

$$\Rightarrow 4\sin^3 x \cos x = (3\sin x - \sin 3x)\cos x$$

$$\int_0^\infty \frac{\sin^4 x}{x^2}\,dx = \int_0^\infty \frac{(3\sin x - \sin 3x)\cos x}{x}\,dx$$

$$= 3\int_0^\infty \frac{\sin x\cos x}{x}\,dx - \int_0^\infty \frac{\sin 3x\cos x}{x}\,dx$$

As, $2\sin x\cos x = \sin 2x$

and, $2\sin 3x\cos x = \sin 4x + \sin 2x$

$$\int_0^\infty \frac{\sin^4 x}{x^2}\,dx = \frac{3}{2}\int_0^\infty \frac{\sin 2x}{x}\,dx - \frac{1}{2}\int_0^\infty \frac{\sin 4x + \sin 2x}{x}\,dx$$

$$\int_0^\infty \frac{\sin(ax)}{x}\,dx = \frac{\pi}{2} \qquad (a > 0)$$

$$\Rightarrow \int_0^\infty \frac{\sin 2x}{x}\,dx = \int_0^\infty \frac{\sin 4x}{x}\,dx = \frac{\pi}{2}$$

$$\boxed{\int_0^\infty \frac{\sin^4 x}{x^2}\,dx = \frac{3}{2}\cdot\frac{\pi}{2} - \frac{1}{2}\left(\frac{\pi}{2}+\frac{\pi}{2}\right) = \frac{\pi}{4}}$$

—— End of Solution 1 ——

Question 2

$$\int_{-\infty}^{\infty} 10\,(x^7 - x^2)^2\, e^{-x^{10}+2x^5}\, dx$$

Proposed by **Atmadeep Sengupta** *(BStat 3rd year)*

First observe that, $-x^{10} + 2x^5 = -\left(x^5 - 1\right)^2 + 1$

$$\Rightarrow e^{-x^{10}+2x^5} = e \cdot e^{-(x^5-1)^2}$$

Also, $x^7 - x^2 = x^2(x^5 - 1)$

$$\Rightarrow (x^7 - x^2)^2 = x^4(x^5 - 1)^2$$

So, $\displaystyle\int_{-\infty}^{\infty} 10\,(x^7 - x^2)^2\, e^{-x^{10}+2x^5}\, dx$

$$= 10e \int_{-\infty}^{\infty} x^4(x^5 - 1)^2 e^{-(x^5-1)^2} dx$$

Substitute, $u = x^5 - 1 \quad \Rightarrow \quad du = 5x^4\,dx$

$$= 10e \int_{-\infty}^{\infty} x^4 (x^5 - 1)^2 e^{-(x^5-1)^2}\,dx$$
$$= 2e \int_{-\infty}^{\infty} u^2 e^{-u^2}\,du$$
$$= 2e \frac{\sqrt{\pi}}{2} \qquad \left[As, \int_{-\infty}^{\infty} u^2 e^{-u^2}\,du = \frac{\sqrt{\pi}}{2}\right]$$
$$= e\sqrt{\pi}$$

$$\boxed{\int_{-\infty}^{\infty} 10\,(x^7 - x^2)^2\, e^{-x^{10}+2x^5}\,dx = e\sqrt{\pi}}$$

—— End of Solution 2 ——

Question 3

Let $f(x) = 3\tan^{-1} x$

g(x)$=\tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$.

$$\int_{-2-\sqrt{3}}^{2+\sqrt{3}} |f(x) - g(x)|\, dx$$

Proposed by Archishman Mukhopadhyay (MStat 2nd year)

$$g(x) = \tan^{-1}\big(\tan(3\tan^{-1} x)\big)$$

$$\left(\text{Using } \tan 3\theta = \frac{3t - t^3}{1 - 3t^2}\right)$$

$$\Rightarrow\; g(x) = \begin{cases} 3\tan^{-1} x, & 3\tan^{-1} x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \\ 3\tan^{-1} x - \pi, & 3\tan^{-1} x \in \left(\frac{\pi}{2}, \frac{3\pi}{2}\right) \\ 3\tan^{-1} x + \pi, & 3\tan^{-1} x \in \left(-\frac{3\pi}{2}, -\frac{\pi}{2}\right) \end{cases}$$

$$3\tan^{-1}x = \pm\frac{\pi}{2} \quad \Rightarrow \quad x = \pm\frac{1}{\sqrt{3}}$$

$$\Rightarrow |f(x) - g(x)| = \begin{cases} 0, & |x| < \dfrac{1}{\sqrt{3}} \\ \pi, & |x| > \dfrac{1}{\sqrt{3}} \end{cases}$$

(Branch cut of $\tan^{-1}\tan y$)

$$\int_{-2-\sqrt{3}}^{2+\sqrt{3}} |f(x) - g(x)|\, dx = \pi\left[\int_{-2-\sqrt{3}}^{-\frac{1}{\sqrt{3}}} dx + \int_{\frac{1}{\sqrt{3}}}^{2+\sqrt{3}} dx\right]$$

$$= 2\pi\left(2 + \sqrt{3} - \frac{1}{\sqrt{3}}\right)$$

$$\boxed{\int_{-2-\sqrt{3}}^{2+\sqrt{3}} |f(x) - g(x)|\, dx = \pi\left(4 + \frac{4}{\sqrt{3}}\right)}$$

— End of Solution 3 —

Question 4

$$\int_1^2 \frac{\lceil x^2 \rceil \{x\}}{\lfloor 2x \rfloor}\, dx$$

Proposed by **Rahul Ganguly (BStat 3rd year)**

$$I = \int_1^2 \frac{\lceil x^2 \rceil \{x\}}{\lfloor 2x \rfloor}\, dx$$

$$\{x\} = x - 1 \qquad (1 \leq x < 2)$$

Discontinuities at $x = \sqrt{2},\ \dfrac{3}{2},\ \sqrt{3}$

$$\Rightarrow [1,2] = [1,\sqrt{2}) \cup [\sqrt{2},\tfrac{3}{2}) \cup [\tfrac{3}{2},\sqrt{3}) \cup [\sqrt{3},2)$$

For $x \in [1, \sqrt{2})$: $\lceil x^2 \rceil = 2, \ \lfloor 2x \rfloor = 2$

$$I_1 = \int_1^{\sqrt{2}} (x-1)\,dx = \frac{(\sqrt{2}-1)^2}{2} = \frac{3}{2} - \sqrt{2}$$

For $x \in [\sqrt{2}, \frac{3}{2})$: $\lceil x^2 \rceil = 3, \ \lfloor 2x \rfloor = 2$

$$I_2 = \frac{3}{2} \int_{\sqrt{2}}^{3/2} (x-1)\,dx = \frac{24\sqrt{2}-33}{16}$$

For $x \in [\frac{3}{2}, \sqrt{3})$: $\quad \lceil x^2 \rceil = 3, \ \lfloor 2x \rfloor = 3$

$$I_3 = \int_{3/2}^{\sqrt{3}} (x-1)\,dx = \frac{15}{8} - \sqrt{3}$$

For $x \in [\sqrt{3}, 2)$: $\quad \lceil x^2 \rceil = 4, \ \lfloor 2x \rfloor = 3$

$$I_4 = \frac{4}{3} \int_{\sqrt{3}}^{2} (x-1)\,dx = \frac{4\sqrt{3}}{3} - 2$$

$$I = I_1 + I_2 + I_3 + I_4$$

$$= \left(\frac{3}{2} - \sqrt{2}\right) + \frac{24\sqrt{2} - 33}{16} + \left(\frac{15}{8} - \sqrt{3}\right) + \left(\frac{4\sqrt{3}}{3} - 2\right)$$

$$\boxed{\int_1^2 \frac{\lceil x^2 \rceil \{x\}}{\lfloor 2x \rfloor}\, dx = -\frac{11}{16} + \frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{3}}$$

—— End of Solution 4 ——

Integration Bee

Round of 4

January 2026

Question 1

$$\int_0^{\pi/2} \frac{x}{3} \sin(4x) \ \tan^{-1}\left(\frac{1+\tan x}{1-\tan x}\right) \ dx$$

Proposed by **Siddhartha Bhattacharya (BStat 3rd year)**

$$\frac{1+\tan x}{1-\tan x} = \tan\left(\frac{\pi}{4}+x\right)$$

$$\tan^{-1}(\tan(\tfrac{\pi}{4}+x)) = \begin{cases} \frac{\pi}{4}+x, & 0 \le x \le \frac{\pi}{4}, \\ x - \frac{3\pi}{4}, & \frac{\pi}{4} \le x \le \frac{\pi}{2} \end{cases}$$

$$I = \frac{1}{3}\left[\int_0^{\pi/4} x(\tfrac{\pi}{4}+x)\sin(4x)\,dx + \int_{\pi/4}^{\pi/2} x(x-\tfrac{3\pi}{4})\sin(4x)\,dx\right]$$

$$= \frac{1}{3}\left[\int_0^{\pi/2} x^2\sin(4x)\,dx + \frac{\pi}{4}\left(\int_0^{\pi/4} x\sin(4x)\,dx - 3\int_{\pi/4}^{\pi/2} x\sin(4x)\,dx\right)\right]$$

$$\int x\sin(4x)\,dx = -\frac{x}{4}\cos(4x) + \frac{1}{16}\sin(4x)$$

$$\int_0^{\pi/4} x\sin(4x)\,dx = \frac{\pi}{16}, \qquad \int_{\pi/4}^{\pi/2} x\sin(4x)\,dx = -\frac{3\pi}{16}$$

$$\int x^2\sin(4x)\,dx = -\frac{x^2}{4}\cos(4x) + \frac{x}{8}\sin(4x) + \frac{1}{32}\cos(4x)$$

$$\int_0^{\pi/2} x^2\sin(4x)\,dx = -\frac{\pi^2}{16}$$

$$I = \frac{1}{3}\left[-\frac{\pi^2}{16} + \frac{\pi}{4}\cdot\frac{10\pi}{16}\right]$$

$$\boxed{\int_0^{\pi/2} \frac{x}{3}\sin(4x)\tan^{-1}\left(\frac{1+\tan x}{1-\tan x}\right)\, dx = \frac{\pi^2}{32}}$$

— End of Solution 1 —

Question 2

$$\int_0^{\pi/2} \frac{\sin^2 x}{\sin^3 x + \cos^3 x}\, dx$$

Proposed by **Jishnu Ray (Bstat 1st year)**

$$I = \int_0^{\pi/2} \frac{\sin^2 x}{\sin^3 x + \cos^3 x}\, dx$$

$$= \int_0^{\pi/2} \frac{\cos^2 x}{\sin^3 x + \cos^3 x}\, dx$$

(By King's Rule)

$$\Rightarrow I = \frac{1}{2} \int_0^{\pi/2} \frac{1}{\sin^3 x + \cos^3 x}\, dx$$

$$= \frac{1}{2} \int_0^{\pi/2} \frac{dx}{(\sin x + \cos x)(1 - \sin x \cos x)}$$

Let $p = \sin x - \cos x$

$$dp = (\sin x + \cos x)\, dx$$

$$p^2 = 1 - 2\sin x \cos x \quad \Rightarrow \quad \sin x \cos x = \frac{1 - p^2}{2}$$

$$1 - \sin x \cos x = \frac{1 + p^2}{2}$$

$$x = 0 \Rightarrow p = -1, \qquad x = \frac{\pi}{2} \Rightarrow p = 1$$

$$\Rightarrow I = \frac{1}{2}\int_{-1}^{1} \frac{dp}{(\sin x + \cos x)^2 \left(\frac{1+p^2}{2}\right)}$$

$$I = 2\int_0^1 \frac{dp}{(1+2\sin x\cos x)(1+p^2)}$$

$$= -2\int_0^1 \frac{dp}{(p^2+1)(p^2-2)}$$

$$= -\frac{2}{3}\int_0^1 \frac{dp}{p^2-2} + \frac{2}{3}\int_0^1 \frac{dp}{p^2+1}$$

$$\boxed{\int_0^{\pi/2} \frac{\sin^2 x}{\sin^3 x + \cos^3 x}\,dx = \frac{\pi}{6} - \frac{\sqrt{2}}{3}\ln(\sqrt{2}-1)}$$

—— End of Solution 2——

Integration Bee

Semi final

January 2026

Question 1

$$\int \ln\left[x^6 + 3x^4 + x^3 - 6x^2 + 27x - 26 - (x^2 + x - 2)^3\right] dx$$

Proposed by **Rahul Ganguly (BStat 3rd year)**

$$x^6 + 3x^4 + x^3 - 6x^2 + 27x - 26$$

$$= (x^6 + 3x^4 + 3x^2 + 1) + (x^3 - 9x^2 + 27x - 27)$$

$$= (x^2 + 1)^3 + (x - 3)^3$$

$$(x^2 + 1)^3 + (x - 3)^3 - (x^2 + x - 2)^3$$

$$= (x^2 + 1)^3 + (x - 3)^3 + (2 - x - x^2)^3$$

Let

$$a = x^2 + 1, \quad b = x - 3, \quad c = 2 - x - x^2$$

$$a + b + c = 0 \Rightarrow a^3 + b^3 + c^3 = 3abc$$

$$(x^2+1)^3+(x-3)^3-(x^2+x-2)^3$$

$$=3(x^2+1)(x-3)(2-x-x^2)$$

$$=3(x^2+1)(x-3)(x+2)(1-x)$$

$$\Rightarrow \ln(\cdots) = \ln 3 + \ln(x^2+1) + \ln(x-3) + \ln(x+2) + \ln(1-x)$$

$$I = x\ln 3 + \int \ln(x^2+1)\,dx + \int \ln(x-3)\,dx$$

$$+\int \ln(x+2)\,dx + \int \ln(1-x)\,dx$$

$$\int \ln(x^2+1)\,dx = x\ln(x^2+1) - 2x + 2\tan^{-1}x$$

$$\int \ln(x-a)\,dx = (x-a)\ln(x-a) - (x-a)$$

$$\boxed{\begin{aligned} I = {} & x\ln 3 + x\ln(x^2+1) - 2x + 2\tan^{-1}x \\ & + (x-3)\ln(x-3) + (x+2)\ln(x+2) \\ & - (1-x)\ln(1-x) - 3x + C \end{aligned}}$$

— End of Solution 1—

Question 2

$$I = \int_0^\infty e^{-2x} \frac{\sin x}{x} \, dx$$

Proposed by **Ayush Baran Sen (BStat 3rd year)**

Define, for $a > 0$: $\quad I(a) = \int_0^\infty e^{-ax} \frac{\sin x}{x} \, dx$

Differentiate w.r.t. a : $\frac{dI}{da} = \int_0^\infty \frac{\partial}{\partial a}\left(e^{-ax}\frac{\sin x}{x}\right) dx = -\int_0^\infty e^{-ax} \sin x \, dx$

Use standard formula: $\int e^{-ax} \sin x \, dx = \frac{e^{-ax}}{a^2+1}(a \sin x + \cos x)$

$$\Rightarrow \frac{dI}{da} = -\frac{1}{a^2+1}$$

Integrate w.r.t. a : $\quad I(a) = -\tan^{-1}(a) + C$

Use $I(0) = \int_0^\infty \frac{\sin x}{x} dx = \frac{\pi}{2} \quad \Rightarrow \quad C = \frac{\pi}{2}$

$$\Rightarrow I(a) = \frac{\pi}{2} - \tan^{-1}(a)$$

$$\boxed{I = I(2) = \frac{\pi}{2} - \tan^{-1}(2)}$$

— End of Solution 2 —

Question 3

$$f(x) = \frac{1-x^2}{1+x^2} + \frac{2x}{1+x^2}i, \qquad g(x) = \sin x$$

$$I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left| \frac{f(x) - (\sqrt{20} - 26i)}{1 - (\sqrt{20} + 26i) f(x)} g(x) \right| dx$$

Proposed by **Rahul Ganguly (B Stat 3rd year)**

$$I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left| \frac{f(x) - (\sqrt{20} - 26i)}{1 - (\sqrt{20} + 26i) f(x)} \right| |g(x)| \, dx$$

Let

$$\alpha = \sqrt{20} - 26i$$

$$I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left| \frac{f(x) - \alpha}{1 - \overline{\alpha} f(x)} \right| |g(x)| \, dx$$

observe that,

$$|f(x)|^2 = \frac{(1 - x^2)^2 + (2x)^2}{(1 + x^2)^2} = \frac{(1 + x^2)^2}{(1 + x^2)^2} = 1$$

$$\Rightarrow |f(x)| = 1$$

Let

$$f(x) = e^{i\theta}, \qquad \alpha = |\alpha| e^{i\phi}$$

$$\left| \frac{f(x) - \alpha}{1 - \overline{\alpha} f(x)} \right| = \left| \frac{e^{i\theta} - |\alpha| e^{i\phi}}{1 - |\alpha| e^{-i\phi} e^{i\theta}} \right|$$

$$= |e^{i\theta}| \left| \frac{1 - |\alpha| e^{i(\phi - \theta)}}{1 - |\alpha| e^{i(\theta - \phi)}} \right|$$

$$= \left| \frac{(1 - |\alpha| \cos(\phi - \theta)) + i |\alpha| \sin(\theta - \phi)}{(1 - |\alpha| \cos(\phi - \theta)) - i |\alpha| \sin(\theta - \phi)} \right|$$

$$= \left| \frac{z}{\overline{z}} \right| = 1$$

$$\Rightarrow I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} |\sin x|\, dx$$

$$\boxed{I = 2}$$

— End of Solution 3 —

Question 4

$$\int_0^{\frac{\pi}{2}} \left[\int_{\frac{\pi}{2}}^{\pi} \frac{\sin x}{x} \, dx \right] dy + \int_{\frac{\pi}{2}}^{\pi} \left[\int_y^{\pi} \frac{\sin x}{x} \, dx \right] dy$$

Proposed by **Ayush Baran Sen (BStat 3rd year)**

Step 1: The first term's inner integral does not depend on y

$$\int_0^{\frac{\pi}{2}} \left[\int_{\frac{\pi}{2}}^{\pi} \frac{\sin x}{x} \, dx \right] dy = \left(\int_0^{\frac{\pi}{2}} dy \right) \left(\int_{\frac{\pi}{2}}^{\pi} \frac{\sin x}{x} \, dx \right) = \frac{\pi}{2} \int_{\frac{\pi}{2}}^{\pi} \frac{\sin x}{x} \, dx$$

Step 2: Consider the second integral

$$\int_{\frac{\pi}{2}}^{\pi}\left[\int_{y}^{\pi}\frac{\sin x}{x}\,dx\right]dy$$

The region of integration is $\frac{\pi}{2}\leq y\leq x\leq\pi$

Reverse the order of integration: $\displaystyle\int_{\frac{\pi}{2}}^{\pi}\int_{\frac{\pi}{2}}^{x}dy\,\frac{\sin x}{x}\,dx=\int_{\frac{\pi}{2}}^{\pi}(x-\frac{\pi}{2})\frac{\sin x}{x}\,dx$

Step 3: Add both contributions

$$\frac{\pi}{2}\int_{\frac{\pi}{2}}^{\pi}\frac{\sin x}{x}\,dx+\int_{\frac{\pi}{2}}^{\pi}(x-\frac{\pi}{2})\frac{\sin x}{x}\,dx=\int_{\frac{\pi}{2}}^{\pi}x\frac{\sin x}{x}\,dx$$

Step 4: Simplify

$$\int_{\frac{\pi}{2}}^{\pi}\sin x\,dx=[-\cos x]_{\frac{\pi}{2}}^{\pi}=1$$

$$\boxed{\int_0^{\frac{\pi}{2}}\left[\int_{\frac{\pi}{2}}^{\pi}\frac{\sin x}{x}\,dx\right]dy+\int_{\frac{\pi}{2}}^{\pi}\left[\int_y^{\pi}\frac{\sin x}{x}\,dx\right]dy=1}$$

— End of Solution 4 —

Integration Bee

Final

January 2026

Question 1

$$\int_0^1 \frac{1-x^{12}}{1+x+x^2}\,\frac{dx}{\ln x}$$

Proposed by **Sidhartha Bhattacharya (B Stat 3rd year)**

$$1 - x^{12} = (1 - x^3)(1 + x^3 + x^6 + x^9)$$

$$1 - x^3 = (1 - x)(1 + x + x^2)$$

$$\Rightarrow \frac{1 - x^{12}}{1 + x + x^2} = (1 - x)(1 + x^3 + x^6 + x^9)$$

$$= (1 - x) + (x^3 - x^4) + (x^6 - x^7) + (x^9 - x^{10})$$

$$I = \sum \int_0^1 \frac{x^p - x^q}{\ln x}\, dx$$

Define

$$F(a) = \int_0^1 \frac{x^a - 1}{\ln x}\, dx$$

Differentiate under the integral sign:

$$F'(a) = \int_0^1 x^a\, dx$$

$$F'(a) = \left[\frac{x^{a+1}}{a+1}\right]_0^1 = \frac{1}{a+1}$$

Integrating with respect to a:

$$F(a) = \ln(a+1) + C$$

Since

$$F(0) = \int_0^1 \frac{1-1}{\ln x}\, dx = 0$$

$$\Rightarrow C = 0$$

$$\boxed{\int_0^1 \frac{x^a - 1}{\ln x}\, dx = \ln(a+1)}$$

$$I = \ln 2 + \ln \frac{5}{4} + \ln \frac{8}{7} + \ln \frac{11}{10}$$

$$= \ln\left(\frac{2 \cdot 5 \cdot 8 \cdot 11}{1 \cdot 4 \cdot 7 \cdot 10}\right)$$

$$\boxed{I = \ln\left(\frac{22}{7}\right)}$$

— End of Solution 1 —

Question 2

$$I = \int_0^1 \left(\frac{x^2 + 2x - 1}{1 + x} \right)^{2026} dx$$

Proposed by **Rahul Ganguly (B Stat 3rd year)**

$$\frac{x^2+2x-1}{1+x} = x - \frac{1-x}{1+x}$$

Define

$$f(x) = \frac{1-x}{1+x}$$

Hence

$$I = \int_0^1 \big(x - f(x)\big)^{2026}\, dx \tag{1}$$

Compute the involution:

$$f(f(x)) = \frac{1-f(x)}{1+f(x)} = x$$

Thus

$$f(f(x)) = x$$

Also,

$$f(0) = 1, \qquad f(1) = 0$$

Let

$$x = f(z)$$

Then

$$dx = f'(z)\,dz$$

From $f(f(z)) = z$,

$$f'(f(z))\,f'(z) = 1$$

Hence

$$f'(z)\,dz = dx$$

Using substitution in (1):

$$I = \int_1^0 \big(f(z) - z\big)^{2026} f'(z)\,dz$$

$$= -\int_0^1 \big(z - f(z)\big)^{2026} f'(z)\,dz \qquad (2)$$

Adding (1) and (2):

$$2I = \int_0^1 (x - f(x))^{2026} (1 - f'(x))\, dx$$

Let

$$y = x - f(x) \quad \Rightarrow \quad dy = (1 - f'(x))\, dx$$

When $x = 0$, $y = -1$
When $x = 1$, $y = 1$

$$2I = \int_{-1}^1 y^{2026}\, dy = 2\int_0^1 y^{2026}\, dy = \frac{2}{2027}$$

$$\boxed{I = \frac{1}{2027}}$$

— End of Solution 2 —

Question 3

$$\int_2^\infty \frac{\lfloor x \rfloor x^2}{x^6 - 1}\, dx$$

Proposed by **Ankit Bhar (B Stat 3rd year)**

Split the integral into unit intervals $[n, n+1]$:

$$\int_2^\infty \frac{\lfloor x \rfloor x^2}{x^6-1}\,dx = \sum_{n=2}^\infty n \int_n^{n+1} \frac{x^2}{x^6-1}\,dx$$

Factor the denominator:

$$x^6 - 1 = (x^3-1)(x^3+1)$$

Using partial fractions,

$$\frac{x^2}{x^6-1} = \frac{1}{2}\left(\frac{x^2}{x^3-1} - \frac{x^2}{x^3+1}\right)$$

Hence,

$$= \frac{1}{2}\sum_{n=2}^\infty n \int_n^{n+1} \left(\frac{x^2}{x^3-1} - \frac{x^2}{x^3+1}\right) dx$$

Observe that

$$\int \frac{x^2}{x^3 \pm 1}\,dx = \frac{1}{3}\log(x^3 \pm 1)$$

Therefore,

$$= \frac{1}{6}\sum_{n=2}^{\infty} n\left[\log\left(\frac{x^3-1}{x^3+1}\right)\right]_n^{n+1}$$

This is a telescoping sum.
To handle boundary terms carefully, write it as a limit:

$$= \frac{1}{6}\lim_{N\to\infty}\sum_{n=2}^{N} n\left[\log\left(\frac{n^3-1}{n^3+1}\right)\right]_n^{n+1}$$

Expanding,

$$= \frac{1}{6}\lim_{N\to\infty}\Bigg(-2\log\frac{2^3-1}{2^3+1} - (3-2)\log\frac{3^3-1}{3^3+1} - \cdots$$

Continuing,

$$= \frac{1}{6} \lim_{N\to\infty} \Big(-2\log\frac{2^3-1}{2^3+1} - \log\frac{3^3-1}{3^3+1} - \cdots - \log\frac{N^3-1}{N^3+1}$$

$$+N\log\frac{(N+1)^3-1}{(N+1)^3+1}\Big)$$

Since

$$\log\frac{(N+1)^3-1}{(N+1)^3+1} \to 0 \quad \text{as } N\to\infty$$

The last term satisfies

$$N\log\frac{(N+1)^3-1}{(N+1)^3+1} \sim N\cdot 2N^{-3} = 2N^{-2} \longrightarrow 0$$

The remaining terms form a telescoping product:

$$-2\log\frac{2^3-1}{2^3+1}-\log\frac{3^3-1}{3^3+1}-\cdots-\log\frac{N^3-1}{N^3+1}$$

Combine the logarithms:

$$=\log\left(\frac{9}{7}\cdot\frac{2^3+1}{2^3-1}\cdot\frac{3^3+1}{3^3-1}\cdots\frac{N^3+1}{N^3-1}\right)$$

Factor each cubic:

$$n^3\pm 1=(n\pm 1)(n^2\mp n+1)$$

Thus,

$$\log\left(\frac{9}{7}\cdot\frac{(2+1)(2^2-2+1)}{(2-1)(2^2+2+1)}\cdot\frac{(3+1)(3^2-3+1)}{(3-1)(3^2+3+1)}\cdots\frac{(N+1)(N^2-N+1)}{(N-1)(N^2+N+1)}\right)$$

This telescopes to

$$=\log\left(\frac{9}{7}\cdot\frac{N(N+1)}{2}\cdot\frac{2^2-2+1}{N^2+N+1}\right)$$

Now take the limit $N\to\infty$:

$$\int_2^\infty \frac{\lfloor x\rfloor x^2}{x^6-1}\,dx=\frac{1}{6}\lim_{N\to\infty}\log\left(\frac{9}{7}\cdot\frac{N(N+1)}{2}\cdot\frac{2^2-2+1}{N^2+N+1}\right)$$

$$=\frac{1}{6}\log\left(\frac{27}{14}\right)$$

$$\boxed{\int_2^\infty \frac{\lfloor x \rfloor x^2}{x^6 - 1}\, dx = \frac{1}{6} \log \frac{27}{14}}$$

— End of Solution 3 —