

# ISI Integration Bee 2026

## Qualification Round — Solution Key

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|    | Question                      | Answer                                |
|----|-------------------------------|---------------------------------------|
| 1. | $\int_0^1 x^3(1-x)^{2026} dx$ | $B(4, 2027) = \frac{3! 2026!}{2030!}$ |

|    | Question                                                          | Answer                          |
|----|-------------------------------------------------------------------|---------------------------------|
| 2. | $\int \frac{2501 \sin^3 x - 2026 \cos^3 x}{\sin^2 x \cos^2 x} dx$ | $2501 \sec x + 2026 \csc x + C$ |

|      | Question                         | Answer                                                                |
|------|----------------------------------|-----------------------------------------------------------------------|
| ★ 3. | $\int \frac{1}{x + x^{2026}} dx$ | $\frac{1}{2025} \ln \left( \frac{x^{2025}}{1 + x^{2025}} \right) + C$ |

|    | Question                                                | Answer          |
|----|---------------------------------------------------------|-----------------|
| 4. | $\int_0^{\pi/2} \frac{1}{\cot \sqrt[3]{2026} x + 1} dx$ | $\frac{\pi}{4}$ |

|    | Question                                                      | Answer                                        |
|----|---------------------------------------------------------------|-----------------------------------------------|
| 5. | $\int \sqrt[2026]{x} \sqrt[2026]{x} \sqrt[2026]{x} \cdots dx$ | $\frac{2025}{2026} x^{\frac{2026}{2025}} + C$ |

|      | Question                                | Answer                                                        |
|------|-----------------------------------------|---------------------------------------------------------------|
| ★ 6. | $\int \frac{1}{(\pi x + e)\sqrt{x}} dx$ | $\frac{2}{\sqrt{\pi e}} \tan^{-1} \sqrt{\frac{\pi x}{e}} + C$ |

|    | Question                                  | Answer                                       |
|----|-------------------------------------------|----------------------------------------------|
| 7. | $\int_0^1 x^3(1-x)^{2026}(1+x)^{2026} dx$ | $\frac{1}{8221512} = \frac{1}{2} B(2, 2027)$ |

|    | Question                                                                                                 | Answer         |
|----|----------------------------------------------------------------------------------------------------------|----------------|
| 8. | $\int_8^{23} \frac{\lfloor x^2 \rfloor^5}{\lfloor x^2 \rfloor^5 + \lfloor x^2 - 62x + 961 \rfloor^5} dx$ | $\frac{15}{2}$ |

|      | Question                                                                           | Answer                                                              |
|------|------------------------------------------------------------------------------------|---------------------------------------------------------------------|
| ★ 9. | $\int_{-e^2}^{e^2} \frac{105x^5 + 4x^4 - \pi x^3 + x^2 - 1.589x + 60}{x^2 + 4} dx$ | $2\left(\frac{4e^6}{3} - 15e^2 + 60 \tan^{-1} \frac{e^2}{2}\right)$ |

|     | Question                                  | Answer          |
|-----|-------------------------------------------|-----------------|
| 10. | $\int_0^\infty \frac{dx}{(1+x^9)(1+x^2)}$ | $\frac{\pi}{4}$ |

|     | Question                                        | Answer                    |
|-----|-------------------------------------------------|---------------------------|
| 11. | $\int_{1/2026}^{2026} \frac{\tan^{-1} x}{x} dx$ | $\frac{\pi}{2} \ln(2026)$ |

|       | Question                                                                        | Answer          |
|-------|---------------------------------------------------------------------------------|-----------------|
| ★ 12. | $\int_0^{\sin^2 x} \arcsin \sqrt{t} dt + \int_0^{\cos^2 x} \arccos \sqrt{t} dt$ | $\frac{\pi}{4}$ |

|     | Question                                     | Answer                                                      |
|-----|----------------------------------------------|-------------------------------------------------------------|
| 13. | $\int \frac{\sin^{2/3} x}{\cos^{14/3} x} dx$ | $\frac{3}{5} \tan^{5/3} x + \frac{3}{11} \tan^{11/3} x + C$ |

|     | Question                                                                | Answer                                                                                                                 |
|-----|-------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------|
| 14. | $\int_0^1 \left[ \frac{2}{x} \right] - 2 \left[ \frac{1}{x} \right] dx$ | $2 - 2\ln 2 = \sum_{k=1}^{\infty} \frac{1}{k(2k+1)} = \sum_{k=1}^{\infty} \left( \frac{1}{k} - \frac{2}{2k+1} \right)$ |

|       | Question                                                      | Answer                        |
|-------|---------------------------------------------------------------|-------------------------------|
| ★ 15. | $\int_1^{16} \tan^{-1} \left( \sqrt{\sqrt{x} - 1} \right) dx$ | $\frac{16\pi}{3} - 2\sqrt{3}$ |