Mathematics Talent Reward Programme (MTRP), 2026

Disclaimer: MTRP is an initiative of the students of Indian Statistical Institute, Kolkata, as a part of their annual techno-cultural-sports fest, Integration, to bolster the love for mathematics among all. This is to clarify that **Indian Statistical Institute itself does NOT conduct this event.**

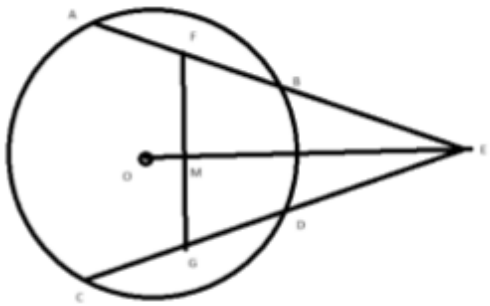
Full Marks: 100 Category: Junior Time: 3 hours

*Each question of **Section A & B** carries **4 points**. Every correct answer with proper justification is awarded **4 points** while every wrong answer leads to **0 point**. A correct answer without justification results in **1 point** being awarded.*

Section A: Objective Questions (MCQs) $(5 \times 4 = 20)$

1. Find the remainder when 45^{4053} is divided by 2027. (2027 is a prime!)
   (A) 1 (B) 45 (C) 2026 (D) None of the above

2. In the figure given below, there is a circle centered at O. AB and CD are two chords extended to intersect each other at E. F and G respectively are the midpoints of AB and CD. OE and FG intersects each other at M. Determine the value of $\angle GFE + \angle FME - \angle FGE$



   (A) $\frac{\pi}{4}$ (B) $\frac{\pi}{2}$ (C)π (D) not constant always

3. Find no. of unordered tuple (a, b, c) with $a, b, c \in \mathbb{N} \cup \{0\}$ such that $2^{a!} + 2^{b!} + 2^{c!}$ is a perfect cube-
   (A) 1 (B) 6 (C)4 (D) 3

4. Jake went into a Christmas Decoration shop and ordered 12 candy canes and 10 jingle bells to decorate his Christmas Tree. Given that each of the items has a positive integer cost, and candy canes cost more than jingle bells, which of the following options is a possible estimate of the total bill?
   (A) 110 (B) 132 (C) 146 (D) 155

5. Suppose there are 100 points on a unit sphere. In how many ways, one can chose 3 points such that they lie on the same hemisphere?
   (A) $\frac{\binom{100}{3}}{6}$ (B) $\frac{\binom{100}{3}}{3}$ (C) $\binom{100}{3}$ (D) $\binom{100}{2}$

Section B: Integer Type $(5 \times 4 = 20)$

1. Suppose N is the number of term with all powers even in $(a_1 + a_2 + \cdots + a_{12})^{24}$. Find the remainder when N is divided by 13.

   $[(a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$ has $1 + 6 + 1 = 8$ terms with all powers even$]$

2. A 24 hour digital clock has 2 faulty digits, that only displays wrong numbers. At one instance the clock showed $19 : 40$ and after 31 minutes the clock showed $00 : 27$. If the actual time at this instance is displayed in the form $AB : CD$, find $(A + B + C + D)$.

3. We define a sequence of distinct characters as $k-super$ if it is the shortest possible sequence that contains each one of the $k!$ permutations of those k characters as a substring. For example, given 3 characters $[A, B, C]$, the sequence $ABCABACBA$ is $3 - super$ with length 9. What is the length of a $4 - super$ sequence?

4. Let ΔABC be a triangle with sides $AB = 6$, $BC = 8$, and $CA = 10$. Let A', B', C' be the reflections of the incentre of ΔABC with respect to AB, BC, and CA respectively.Let the area of triangle $\Delta A'B'C'$ is $\dfrac{m}{n}$ where $gcd(m, n) = 1$. Find $|m - n|$.

5. Let $P(x)$ be a real polynomial of degree 2026 satisfying

$$(\cos^{2026}\ \theta)P(\tan\theta) = (\sin^{2026}\ \theta)P(\cot\theta) \quad \forall\ \theta \in (0, \frac{\pi}{2})$$

   Find the product of roots of $P(x)$.

Section C: Subjective Problems $(6 \times 10 = 60)$

1. **A Curious Discovery**
   One day in a mathematics class, Atmadeep asked his friends to try a small number experiment. He told them to choose **eight consecutive positive integers**. Each student squared their number, and then all the squared numbers were added together. Surprisingly,

   > Every time, the total of the eight squared numbers was equal to **four times a prime number**.

   Then Atmadeep became curious and studied this pattern more carefully. After checking many cases, he discovered a surprising rule:

   > Whenever the sum is of the form $4p$, with p prime, the number $p + 1$ is always divisible by 12.

   Can you explain Atmadeep why the number $p + 1$ must always be divisible by 12?

2. **Shortage of roots**
   Suppose $P(x)$ is an integer polynomial with degree d. Prove that, $P(x)^2 - 1$ has atmost $d + 2$ many distinct integer roots.

3. **A Boring Afternoon with Cards**
   On a boring afternoon, having nothing to do, Ahan and Shounak decide to play a game. They collect some cards and arrange them into four piles, each containing 20 cards.

   - Ahan carries the cards from one pile to another **one by one**.
   - If Ahan transfers a card from pile A to pile B, Shounak rewards him with an amount of money equal to

     $$\text{(number of cards in pile A)} - \text{(number of cards in pile B)}$$

     where the numbers are counted **just before the transfer**.

- If the number is negative, Ahan must pay the corresponding amount to Shounak. (Shounak is generous enough to allow Ahan to pay later if he runs out of money.)
- If at any moment all four piles contain the **same number of cards**, Shounak may stop the game immediately, or allow it to continue.

After 2026 moves, the game stops. Find the maximum and minimum possible **net income** of Ahan at the end of the game over all possible sequences of moves.

4. **Patterns in product of primes**
Show that

$$\prod_{\substack{p \leq n \\ p \text{ prime}}} p < 4^n$$

for every $n \in \mathbb{N}$.

5. **A Clever Challenge**
Aryanil writes n positive real numbers $x_1, x_2, ..., x_n$ on a blackboard and challenges Arnab to write n reals $y_1 \geq y_2 \geq ... \geq y_n \geq 0$ such that for all $k \in \{1, 2, ..., n\}$

$$\sum_{i=1}^{k} y_i \leq \sum_{i=1}^{k} x_i$$

Arnab wins the challenge if

$$\sum_{i=1}^{n} y_i^2 > \sum_{i=1}^{n} x_i^2$$

Can Arnab ever win? (assuming that Aryanil chooses the numbers randomly and has no particular strategy)

6. **The Magical Protection Circle**
A wizard is protecting trees in a magical forest. He can draw a **magic protection circle** on the ground. Every tree **inside the circle is protected** from monsters. Every tree **outside the circle is not protected**. The wizard wants to draw the circle so that it protects **exactly the trees he chooses**.
Part 1: Three Trees
Suppose three trees grow in the forest, and they are **not standing in a straight line**.The wizard may decide that any combination of trees should be protected. For example he can protect all three trees, protect only one tree, protect two trees but not the third or protect none of them.
Can the wizard always draw a circle so that exactly the trees he chooses are inside the circle?
Part 2: Four Trees
Now suppose there are **four trees** growing in the forest (not any three of them are in a straight line). Again, the wizard can choose any trees to protect. Is it always possible for the wizard to draw a circle that protects exactly the trees he wants? Or is there some choice of trees that makes this impossible? Finally what is the **largest number of trees** for which the wizard can always draw a circle that protects **any chosen combination of trees**? [3+7 = 10]