



MATHEMATICS TALENT REWARD PROGRAMME (MTRP), 2026

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Full Marks: 100

Category: Senior

Answer Key & Solutions

Multiple Choice Questions ($3 \times 8 = 24$)

1. (D) 3

Let $x^3 = 2^a + 2^b + 2^c$. First see that no solutions exist when $a, b, c \geq 3$ (check $x \equiv 0 \pmod{3}$ but $x^3 \equiv 3 \pmod{9}$). A similar thing happens when two of them are ≥ 3 or only one of them is. So, $a, b, c \leq 2$ must hold. Just check that $2 + 2 + 4 = 8$ is the only way to get a perfect cube then. This can happen only when (a, b, c) is some permutation of $(1, 1, 2)$, $(0, 1, 2)$ or $(0, 0, 2)$.

2. (B) 98

Let $y = 1 + e^x$. $Q(y) + Q(2 - y) = Q(2)$ holds infinitely many y , so it holds for every $y \in \mathbb{R}$. Differentiate w.r.t. y , $Q'(y) = Q'(2 - y)$. Let $Q(x) = a_0 + a_1x + \dots + a_{99}x^{99}$. Just differentiate term and term and consider only the leading two or three coefficients. Equate them on both sides.

3. (D) Both (B) and (C) are true

$|z_1 - z_2| + \dots + |z_{2026} - z_1| < \text{arc}(z_1 z_2) + \dots + \text{arc}(z_{2026} z_1) = \text{circumference of unit circle} = 2\pi$ (equality can never occur). Similarly, $|z_1^2 - z_2^2| \leq |z_1 - z_2||z_1 + z_2| < \text{arc}(z_1 z_2)(|z_1| + |z_2|) = 2\text{arc}(z_1 z_2)$.

4. (A) 6

Note that $FE \parallel BD$. And Thus by Ceva, extended AK will cut BD at mid-point of BD , centre of the parallelogram, say, O . Thus, K is on the diagonal of the parallelogram. By Menelaus, $OK : KA = 1 : 4 \implies OK = 1 \implies CK = 6$

5. (A) 1

$f(n) = \sum_{d|n} \frac{1}{d} = \frac{1}{n} \sum_{d|n} \frac{n}{d} = \frac{1}{n} \sum_{d|n} d = \frac{\sigma(n)}{n}$ where $\sigma(n)$ is the sum of positive integer divisors of n . Now, $f(n) = 2 \iff \sigma(n) = 2n$ where we can set $n = p_1 p_2 \dots p_k$ (all distinct primes) since n is also square-free. It's easy to see that $\sigma(n) = (p_1 + 1)(p_2 + 1) \dots (p_k + 1)$. Just equate them and check that $k = 2, p_1 = 2, p_2 = 3$ is the only option ($n = 6$).

6. (A) 9

The largest possible value of x is attained when the 2026-digit number divisible by 9 is $999 \dots 9$. So, $x \leq 18234$. Among these values of x , y is maximum at 36 when $x = 9999$. Now, among $y \in \{9, 18, 27, 36\}$, there is only one possible value for z which is 9.

7. (A) If $g(x)$ has a root in (a, b) , it must have at least two roots in (a, b)

Consider $g(x) = \sin x$ and $a = 0, b = 2\pi$.

They don't need to justify why the other options are correct. If they do so, it's fine. Note that $g(a) = g(b)$ is a must. If $g(a) = 0$, so is $g(b) = 0$. If neither of them are zero, you can try to draw a graph and show why g can't have exactly one root in (a, b) and still have zero area. $\int_{\theta}^{\theta+1} f(x)dx = 0 \implies f(\theta + 1) = f(\theta)$ when f is continuous. $f(x) = \sin 2\pi x$ is a non-trivial example of this.

8. (B) 27

Check modulo 3 to see that at least one of them must be divisible by 3. If $p = 3, q^2 + r^2 = 18 \implies q = r = 3$ (since they must be divisible by 3). If not, then $q = 3$ without loss of generality. Then, $p^3 = p^2 + r^2 + 9$. Clearly, $p = 2$ or $r = 2$ doesn't work. Then, check that $p \equiv -1 \pmod{4}$ is the only possibility. Use Fermat's theorem. There are no other solutions.

Integer Type Questions ($4 \times 4 = 16$)

1. Answer: $\boxed{200}$

Every path can be reflected about the diagonal to get another such path. So, on an average, the area is half of the total area of the square.

2. Answer: $\boxed{1013}$

We can show that $f(n) = \frac{2}{n(n+1)}f(1)$ for every $n \geq 1$ using induction on n . Otherwise, we can solve the recursion as follows: $f(1) + \dots + f(n-1) + f(n) = (n-1)^2 f(n-1) + f(n) = n^2 f(n) \implies f(n) = \frac{n-1}{n+1} f(n-1)$. Iterate upto $n = 1$.

3. Answer: $\boxed{2}$

See that $a_{n+1} = 2 + 1/a_n$. Let $a_n = p_n/q_n$ in reduced form. Then, $q_{n+1} = p_n$ and $p_{n+1} = 2p_n + p_{n-1} \implies$ the characteristic polynomial for p_n is $x^2 - 2x - 1$. Just check that $p_n = \frac{1}{2} \left((1 + \sqrt{2})^{n+1} + (1 - \sqrt{2})^{n+1} \right)$. Using $q_n = p_{n-1}$ and then taking limit at $n \rightarrow \infty$, you get the answer to be $\lfloor 1 + \sqrt{2} \rfloor = 2$.

Note that we can answer this without solving the recursion, assuming that $\lim_n a_n$ exists and equals ℓ , one can solve $\ell = 2 + 1/\ell$ for ℓ and then argue why ℓ is not negative.

4. Answer: $\boxed{14}$.

Let the coordinates of Jerry be $(\cos \theta, \sin \theta)$ where $\theta \in [0, 2\pi]$ and the coordinates of Tom be $(x, 0)$. Note that x and θ are both functions of t and at $t = 0, x(0) = -1, \theta(0) = 0$. Horizontal velocities are calculated (consider direction here, because we use angular coordinates).

$$-\sin \theta \frac{d\theta}{dt} = -\frac{1}{2} \frac{dx}{dt} \implies dx = 2 \sin \theta d\theta \implies x = C - 2 \cos \theta.$$

To find C , set $\theta = 0$ and $x = -1$ and $C = 1$. Then, minimize the distance $\sqrt{(1 - 3 \cos \theta)^2 + \sin^2 \theta}$ with respect to θ . Easy thing to do is to take the square of this and differentiate to get the minimizing value $7/8$. Then, minimum distance is $\frac{\sqrt{14}}{4}$ and $m = 14$.

Subjective Type Questions ($12 \times 5 = 60$)

1. Wes has a winning strategy. To prove it, we provide a winning strategy for Wes: set $c = 1$ and however Paul chooses one of the two remaining coefficients a and b , Wes chooses the other one in such a way that $a + b = -3$. This way, $p(0) = 1, p(1) = -1$ and $\lim_{x \rightarrow \infty} p(x) = \infty$ and $\lim_{x \rightarrow -\infty} p(x) = -\infty$. By IVT, $p(x) = 0$ has at least one root in each of the three intervals $(-\infty, 0), (0, 1)$ and $(1, \infty)$ and since it is a degree 3 polynomials, this guarantees all three roots are real and distinct.

2. Let $F(x) = \int_a^x f(t)dt$ and $G(x) = \int_a^x g(t)dt$. Note that F, G are both differentiable on $[a, b]$ with $F'(x) = f(x)$ and $G'(x) = g(x)$. Let $H(x) = G(x)/F(x)$ for every $x \in (a, b]$. Note that H is not well-defined for $x = a$. We extend the definition of $H(x)$ to $[a, b]$ by setting $H(a) = \lim_{x \rightarrow a} \frac{G(x)}{F(x)} = \lim_{x \rightarrow a} \frac{G'(x)}{F'(x)} = \frac{g(a)}{f(a)} = 0$. Now, H is continuous on $[a, b]$ and clearly differentiable on (a, b) and $H(a) = 0 = H(b)$. By Rolles' theorem on H , we see that there exists some $c \in (a, b)$ such that $H'(c) = 0 \implies f(c)G(c) = g(c)F(c)$.

3. Let the players' scores be $s_1 > s_2 > s_3 > \dots > s_n$. Since each pair of players plays exactly one game, the total number of games is $\binom{n}{2}$. Each game contributes exactly 1 point in total, hence

$$\sum_{i=1}^n s_i = \binom{n}{2} = \frac{n(n-1)}{2}.$$

The winner can score at most $n - 1$ points, so $s_1 \leq n - 1$. Therefore

$$\sum_{i=2}^n s_i = \frac{n(n-1)}{2} - s_1 \geq \frac{n(n-1)}{2} - (n-1) = \frac{(n-1)(n-2)}{2}.$$

Thus the average score of the remaining $n - 1$ players is at least $\frac{n-2}{2}$. Assume for contradiction that $s_2 + s_3 \leq s_1$. Since all scores are distinct multiples of $\frac{1}{2}$, we must have

$$s_2 \geq s_3 + \frac{1}{2}.$$

Hence

$$2s_3 + \frac{1}{2} \leq s_2 + s_3 \leq s_1 \leq n - 1 \implies s_3 \leq \frac{2n-3}{4}.$$

Because the scores are strictly decreasing and differ by at least $\frac{1}{2}$,

$$s_4 \leq s_3 - \frac{1}{2}, \quad s_5 \leq s_3 - 1, \quad \dots$$

Summing the largest possible values consistent with these inequalities give

$$\sum_{i=2}^n s_i < \frac{(n-1)(n-2)}{2}$$

for $n \geq 5$, which contradicts the earlier lower bound

$$\sum_{i=2}^n s_i \geq \frac{(n-1)(n-2)}{2}.$$

4. Use Fermat's Little theorem to reduce the equation modulo p . We get $a + b \equiv c + d \pmod{p}$. This implies $p \mid (a - c) + (b - d)$ and using triangle inequality $|a - c| + |b - d| \geq |a - c + b - d|$, the lower bound being a non-negative multiple of p . Now, if this multiple is non-zero, then it must be $\geq p$. Consider the case when $a - c + b - d = 0$.

Define $f(x) = x^p$ which is a differentiable function. First note that since $a - c = d - b$ and these are all distinct positive integers, $a > c > b > d$ can be assumed without loss of generality. Divide the two equations you have and apply Lagrange's mean value theorem, on $[c, a]$ and $[b, d]$:

$$\frac{a^p - c^p}{a - c} = \frac{d^p - b^p}{d - b} \implies px_1^{p-1} = px_2^{p-1} \implies x_1 = x_2,$$

where $x_1 \in (c, a)$ and $x_2 \in (b, d)$ (contradiction!).

5. Claim : O lies on (APQ) .

Proof: $\angle POQ = 360^\circ - B - C - 2A = 180^\circ - A$

Claim : E lies on (ABC) .

Proof: Join Q, E Consider $\triangle QOE$ and $\triangle QOC$. Note that $\angle QOE = C = \angle QOC$ and $\angle OEQ = 90^\circ - B = \angle OCQ$ so, $\triangle QOC \cong \triangle QOE$ and so, $OE = OC$, Thus E lies on (ABC)

Claim: $AEBC$ is cyclic trapezium.

Proof: $\angle COE = 2C, OC = OE$, so, $\angle OEC = 90^\circ - C \implies \angle QEC = B - C = \angle QCE = B - C \implies \angle ECB = B$. Thus we conclude that $AEBC$ is a cyclic trapezium.

Claim : B is reflection of E about OP and C is the reflection of F about OQ .

Proof: Again note that $\triangle OPE$ and $\triangle OPB$ are congruent. Our claim follows from this observation.

Claim : F is reflection of E about PQ .

Proof: By Simson's line as, O, P, Q, E are concyclic, Reflection of E about PQ must lie on BC by previous claim. So, proved.

So, $QE = QF = QC \implies \angle QFC = C \implies Q, O, F, C$ are concyclic. Similarly, P, O, F, B are concyclic. Now note that $FO \perp PQ$ and $\angle PFO = \angle PQO = 90^\circ - C; \angle OFQ = \angle OPQ = 90^\circ - B; \angle OPF = \angle OQF = 90^\circ - A$.

Hence, conclude that O is the orthocentre of $\triangle FPQ$.

End of Paper