Mathematics Talent Reward Programme (MTRP), 2026

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1 Section A: Objective

Question 1

Solution: The equation given is $x^x = 2^{64}$. To solve for x, we need to express the right-hand side in the form of y^y. By utilizing the laws of exponents, we can rewrite 2^{64} as $(2^4)^{16}$. Since $2^4 = 16$, the expression becomes 16^{16}. Comparing both sides of the equation $x^x = 16^{16}$, it is clear that $x = 16$. The correct option is (B).

Question 2

Solution: Let the number of 5-rupee coins be x and the number of 10-rupee coins be y. The initial value of the collection is given by the equation $5x + 10y = 140$, which simplifies to $x + 2y = 28$. When the coins are swapped, the new value gives the equation $10x + 5y = 160$, simplifying to $2x + y = 32$. Adding these two simplified equations together yields $3x + 3y = 60$. Factoring out the 3 gives $3(x + y) = 60$, which directly leads to $x + y = 20$. Therefore, the total number of coins is 20. The correct option is (C).

Question 3

Solution: The problem requires evaluating $a^3 + b^3$, which can be factored using the algebraic identity $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$. We are given $a + b = 5$ and $a^2 + b^2 = 17$. To complete the substitution, we need the value of ab. Using the identity $(a + b)^2 = a^2 + b^2 + 2ab$, we substitute the known values to get $25 = 17 + 2ab$. Solving this gives $2ab = 8$, so $ab = 4$. Substituting all these values into our first identity gives $a^3 + b^3 = 5 \times (17 - 4)$, which evaluates to $5 \times 13 = 65$. The correct option is (C).

Question 4

Solution:
By adding and subtracting three odd and two even numbers, we cannot get an even number. Therefore, some of the operations must be multiplication or division.
Because each number is greater than the previous one, any division would give us a fraction. We could make an integer out of a fraction if we multiplied the fraction by a number divisible by the denominator. In this case, the only possibility would be to divide by 2 and multiply by 3 and 4. But

$$1 \div 2 \times 3 \times 4 = 6,$$

which makes it impossible to obtain 0 by adding or subtracting 5.
So, we cannot use any divisions but must use at least one multiplication. After testing various possibilities, we find that the only way to get 0 is

$$1 \times 2 - 3 - 4 + 5 = 0.$$

So, the first operation must be multiplication.

Question 5

Solution: Let the terms of the geometric progression be x, $y = xr$, and $z = xr^2$. The problem states that the sequence $x, 2y, 3z$ forms an arithmetic progression. According to the properties of an arithmetic progression, twice the middle term equals the sum of the first and third terms, giving $2(2y) = x + 3z$. Substituting the geometric terms into this equation yields $4xr = x + 3xr^2$. Since the problem specifies that the terms are non-zero, we can divide the entire equation by x to obtain $4r = 1 + 3r^2$, which rearranges to the quadratic equation $3r^2 - 4r + 1 = 0$. Factoring this quadratic gives $(3r - 1)(r - 1) = 0$. The roots are $r = 1$ and $r = 1/3$. Since the terms of the geometric progression are distinct, r cannot be 1. Therefore, the common ratio must be 1/3. The correct option is (B).

Question 6

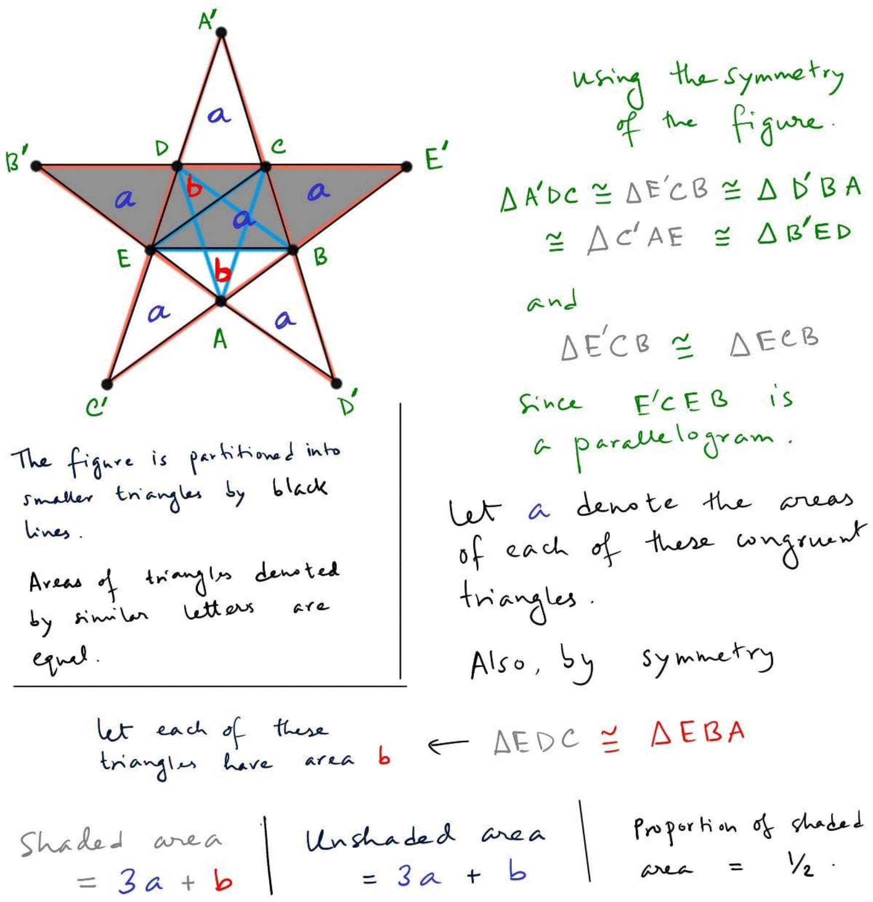
Solution: Let us define an auxiliary polynomial $Q(x) = P(x) - x$. Since $P(x)$ is given as a monic polynomial of degree 4, $Q(x)$ will also be a monic polynomial of degree 4. The given conditions $P(1) = 1$, $P(2) = 2$, $P(3) = 3$, and $P(4) = 4$ imply that $Q(x)$ equals zero at exactly $x = 1, 2, 3$, and 4. Consequently, we can write $Q(x)$ in its factored form as $(x - 1)(x - 2)(x - 3)(x - 4)$. To find the value of $P(5)$, we evaluate $Q(5)$ and add 5. Plugging in $x = 5$ gives $Q(5) = (4)(3)(2)(1) = 24$. Thus, $P(5) = 24 + 5 = 29$. The correct option is (C).

Question 7

Solution: We need to find the number of 3-digit positive integers whose digits sum to exactly 6. Let the number be represented by the digits a, b, c. We are looking for the number of integer solutions to the equation $a + b + c = 6$, with the constraints that $a \geq 1$ (since it is a 3-digit number) and $b, c \geq 0$. To simplify, we can introduce a new variable $a' = a - 1$, where $a' \geq 0$. Substituting this into our equation gives $a' + b + c = 5$. Using the stars and bars combinatorial method, the number of non-negative integer solutions is given by the combination $\binom{5+3-1}{3-1}$, which simplifies to $\binom{7}{2}$. Calculating this yields $(7 \times 6)/2 = 21$. The correct option is (C).

Question 8

Solution:



Question 9

Solution: Let the original principal amount be P and the annual compound interest rate be R. We are given that the interest earned strictly during the second year is Rs. 1200, and strictly during the third year is Rs. 1440. Under compound interest, the interest earned in any given year is simply the interest from the previous year multiplied by the compounding factor $(1+R)$. Therefore, $1200 \times (1+R) = 1440$. Solving this gives $1+R = 1.2$, meaning the interest rate R is 20%, or 0.2. The interest for the second year is calculated as the principal plus the first year's interest, all multiplied by the rate: $P(1+R)^1 \times R = 1200$. Substituting our knowns gives $P(1.2)(0.2) = 1200$, which simplifies to $P(0.24) = 1200$. Solving for P yields 5000. The correct option is (B).

Question 10

Solution: Let us assume we take an equal weight of 100 units from both metallic alloys to simplify calculations. In the first alloy, the ratio of gold to copper is 2:3, meaning gold constitutes 2/5 of the total. So, the first alloy contributes $\frac{2}{5} \times 100 = 40$ units of gold. In the second alloy, the ratio is 3:7, meaning gold is 3/10 of the total. Thus, the second alloy contributes $\frac{3}{10} \times 100 = 30$ units of gold. When these equal weights are melted together, the new completely mixed alloy has a total weight of 200 units. The total amount of gold in this mixture is $40 + 30 = 70$ units. The percentage of gold is therefore $\frac{70}{200} \times 100\% = 35\%$. The correct option is (B).

Section B: Subjective

Problem 1 (10 Marks)

(a) The prime numbers in ascending order are $2, 3, 5, 7, 11, 13, 17, 19, 23, 29, \ldots$
Therefore, $p_1 = 2$ and $p_{10} = 29$.

(b) We need $p_{k+2} - p_{k+1} = p_{k+1} - p_k = 2$. This implies p_k, p_{k+1}, p_{k+2} are three consecutive primes forming an arithmetic progression with a common difference of 2. The only such prime triplet is $(3, 5, 7)$. Since $p_2 = 3$, $p_3 = 5$, and $p_4 = 7$, the smallest k is 2.

(c) We are given $p_k = \dfrac{p_l + p_{l+1}}{2}$. This means p_k is the arithmetic mean of two consecutive primes, p_l and p_{l+1}. Because $p_l < p_{l+1}$, their average must lie strictly between them: $p_l < p_k < p_{l+1}$. However, by definition, there are no prime numbers strictly between two consecutive prime numbers p_l and p_{l+1}. Thus, there is **no such** k.

(d) We are given $p_k = \sqrt{p_l \cdot p_{l+1}}$, which implies $p_k^2 = p_l \cdot p_{l+1}$. Since p_l and p_{l+1} are distinct prime numbers, their product $p_l \cdot p_{l+1}$ has exactly four divisors $(1, p_l, p_{l+1}, p_l p_{l+1})$ and cannot be the square of an integer. Alternatively, $p_l < \sqrt{p_l \cdot p_{l+1}} < p_{l+1}$, and no prime can exist between two consecutive primes. Thus, there is **no such** k.

Problem 2 (10 Marks)

(a) The system is: (1) $2x + 3y = 6$ (2) $3x + 4.5y = 9$
Multiplying equation (1) by 1.5 yields $3x + 4.5y = 9$, which is exactly equation (2). Since the equations are identical, the system is dependent and has infinitely many solutions. The solutions can be represented as $(x, y) = (x, 2 - \frac{2}{3}x)$ or $(3 - 1.5y, y)$ for any real numbers x or y.

(b) The system is: (1) $3x + 5y = 25$ (2) $6x + 10y = 45$
Multiplying equation (1) by 2 yields $6x + 10y = 50$. Subtracting this from equation (2) gives $0 = -5$, which is a contradiction. Therefore, the lines are parallel and there are no solutions.

Problem 3 (10 Marks)

Let the number of lotuses Akash plucks initially be x. Let the number of lotuses he gives to each person (Pat, Misty, and Shreja) be y.

- ☐ **First Crossing:** Lotuses double to $2x$. He gives y to Pat. Remaining $= 2x - y$.
- ☐ **Second Crossing:** Lotuses double to $2(2x - y) = 4x - 2y$. He gives y to Misty. Remaining $= 4x - 3y$.

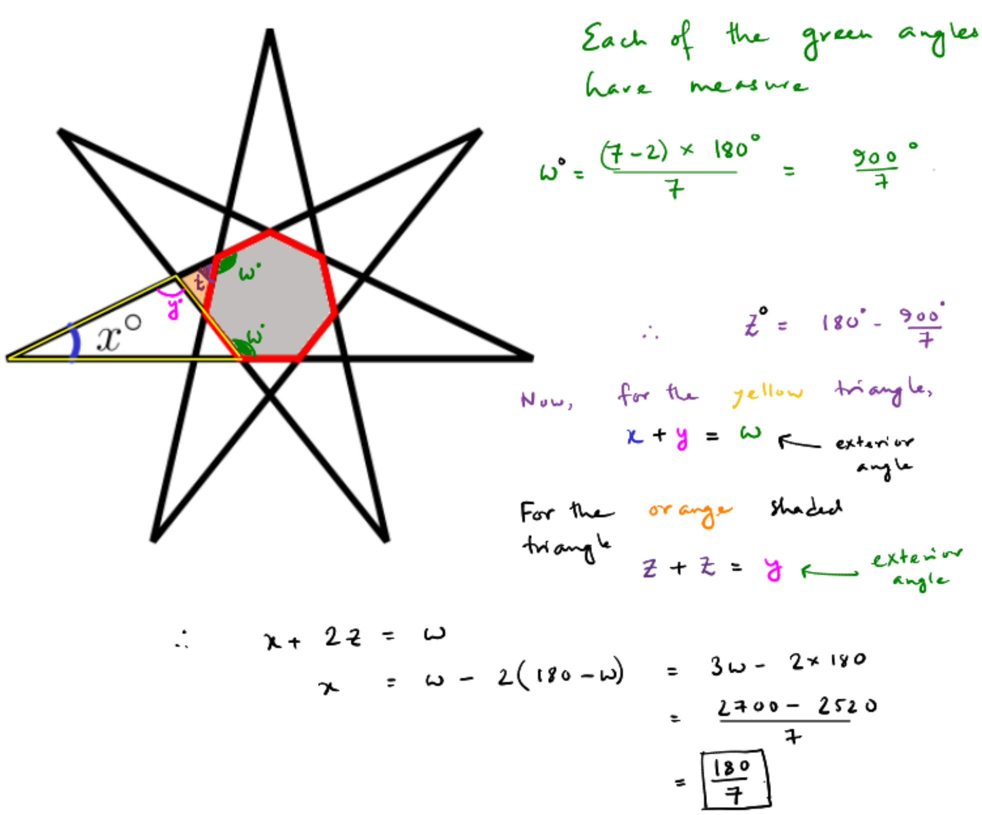
- ☐ **Third Crossing:** Lotuses double to $2(4x-3y) = 8x-6y$. He gives y to Shreja. Remaining $= 8x - 7y$.

We are given that he is left with 0 lotuses. Therefore, $8x - 7y = 0 \implies 8x = 7y \implies \frac{x}{y} = \frac{7}{8}$. Since x and y must be whole integers (no fractions allowed), the smallest possible natural numbers satisfying this ratio are $x = 7$ and $y = 8$. Minimum lotuses plucked initially $= 7$. Lotuses Shreja receives $= 8$.

Problem 4 (10 Marks)

E makes a statement with a clear consequence, so we'll hypothesize about his identity. Suppose **E** is a human. Then **C** is a werewolf. For his statement to be a lie, we can have any number of humans other than two. Since **C** and **E** are different, **B** statement is a lie, so **B** is a werewolf. That makes **A** a werewolf as well. Then **D**'s statement is true, so **D** is a human. However, that means there are two humans, which contradicts **C**'s lie. So, our original hypothesis that **E** is a human is false — he must instead be a werewolf. As a werewolf, **E** is lying, so **C** is a human. That means there are exactly two humans and thus three werewolves. But we'll still check that this is consistent with everyone's statement and identity. **C** and **E** are different, so **B** is a werewolf. Then **B** and **E** are the same, so **A** is a human. **A** and **B** are different, so **D** is a werewolf. Everyone's statement matches their identity, and we indeed have three werewolves **B**, **D** and **E**.

Problem 5 (10 Marks)



Alternate Solution:
If we trace the star with a pen, starting from any point on a side of the star, each time the pen reaches a vertex, it changes direction by an angle of $(180 - x)^\circ$. An argument along these lines can fetch the full solution.

Problem 6 (10 Marks)

(a) Let X and Y be two good numbers. By definition, they can be written as $X = a^2 + b^2$ and $Y = c^2 + d^2$ for some natural numbers a, b, c, d. Their product is $XY = (a^2+b^2)(c^2+d^2)$. Expanding this gives: $a^2c^2 + a^2d^2 + b^2c^2 + b^2d^2$. Using the hint, we expand $(ac+bd)^2 + (ad-bc)^2$: $(ac+bd)^2 + (ad-bc)^2 = (a^2c^2 + 2abcd + b^2d^2) + (a^2d^2 - 2abcd + b^2c^2) = a^2c^2 + b^2d^2 + a^2d^2 + b^2c^2$. This proves Brahmagupta's Identity: $(a^2+b^2)(c^2+d^2) = (ac+bd)^2 + (ad-bc)^2$. Since the product is expressed as the sum of two squares, the product XY is also a good number.

(b) We need the smallest natural number expressible as the sum of two non-zero squares in two ways. Checking sums of squares systematically: $50 = 7^2 + 1^2 = 5^2 + 5^2$. (Note: $65 = 8^2 + 1^2 = 7^2 + 4^2$ is also valid but not the smallest). The smallest such number is 50.

(c) We need a good number expressible in at least 3 ways. We can multiply good numbers that have multiple representations. Let's use 65 (which is $8^2 + 1^2 = 7^2 + 4^2$) and multiply by another good number, say $5 = 2^2 + 1^2$. $325 = 65 \times 5$. Using $65 = 8^2 + 1^2$ and $5 = 2^2 + 1^2$ with the identity $(ac+bd)^2 + (ad-bc)^2$: Way 1: $(8(2)+1(1))^2 + (8(1)-1(2))^2 = 17^2 + 6^2 = 325$. Way 2: $(8(1)+1(2))^2 + (8(2)-1(1))^2 = 10^2 + 15^2 = 325$. Using $65 = 7^2 + 4^2$ and $5 = 2^2 + 1^2$: Way 3: $(7(2)+4(1))^2 + (7(1)-4(2))^2 = 18^2 + (-1)^2 = 18^2 + 1^2 = 325$. Therefore, 325 is a valid answer. (1105 and other multiples are also acceptable).