SPARQ Junior 2026 Question Paper

Time: 120 minutes **Date: January 03, 2026**

Question 1

StreamLine, a video streaming platform with ten lakhs registered accounts, is currently evaluating a new pricing strategy to address high server costs. Under the current free, ad-supported model, the company earns Rs. 80 per hour in ad revenue against a streaming cost of Rs. 50 per hour, resulting in a net profit of Rs. 30 per hour watched. Aggregate usage data shows that the global average viewing time across all accounts is 2.0 hours per month, yet 50% of the total accounts show exactly zero activity. However, internal analysis reveals that this data is skewed by a "zero-inflated" user structure: 40% of the total accounts are "Ghost" users who have abandoned the platform and never watch content, while the remaining population consists of "Active" users who generally watch content but may occasionally register zero hours. The Chief Marketing Officer has proposed replacing the ad model with a flat Rs. 500 monthly subscription fee, featuring a "Data Saver Refund" where any user who watches exactly zero hours receives a full 100% refund. To evaluate the viability of this proposal, you must first use the provided global data to mathematically derive the average viewing time of the "Active" segment and the specific probability that an "Active" user watches zero hours. Using these derived figures, calculate the expected profit per account under the new model—accounting for the fact that zero-hour users generate no revenue and incur no costs—and determine if this strategy yields a higher expected profit than the current average of Rs. 60 per user.

1.1 Based on the usage data provided for **StreamLine**, calculate the average viewing time (in hours per month) of an *Active* user.

(A) 2.0

(B) 3.0

(C) 3.33

(D) 5.0

Answer: (C)

1.2 What is the probability that an *Active* user watches exactly 0 hours in a month?

(A) $\frac{1}{10}$

(B) $\frac{1}{6}$

(C) $\frac{1}{4}$

(D) $\frac{1}{3}$

Answer: (B)

1.3 Under the proposed subscription model with a full refund for zero usage, what is the expected profit per account (in rupees) for StreamLine?

(A) 60

(B) 100

(C) 150

(D) 50

Answer: (C)

1.4 Based on the expected profit calculations, which of the following statements is correct?

(A) The new subscription model is less profitable than the current ad-based model.

(B) The new subscription model yields the same expected profit as the current model.

(C) The new subscription model is more profitable than the current ad-based model.

(D) The profitability of the new model cannot be determined from the data.

Answer: (C)

1.5 Assuming that the behaviour of *Active* users remains unchanged, what is the *maximum percentage of Ghost users* the platform can have before the new subscription model becomes less profitable than the benchmark profit of Rs. 60 per account?

(A) 30%

(B) 36%

(C) 40%

(D) 50%

Answer: (B)

Question 2

2.1 Determine the value of Δ_1.

(A) $(a-b)(b-c)(c-a)$

(B) $(a-b)(b-c)(c-a)(a+b+c)$

(C) $(a-b)(b-c)(c-a)(a^2+b^2+c^2)$

(D) 0

Answer: (B)

2.2 Which of the following statements is true about $\Delta_2(\lambda)$?

(A) $\det \Delta_2(\lambda)$ depends on λ

(B) $\det \Delta_2(\lambda)$ is linear in λ

(C) $\det \Delta_2(\lambda)$ is independent of λ

(D) $\det \Delta_2(\lambda) = 0$

Answer: (C)

2.3 Which of the following best describes the nature of the function $f(x)$?

(A) a constant function

(B) a linear function

(C) a quadratic function

(D) any polynomial

Answer: (B)

2.4 Which of the following expressions correctly represents Δ_3?

(A) $(x+a+b)\big(x^2+a^2+b^2-ab-bx-ax\big)$

(B) $(x-a-b)^3$

(C) $x^3+a^3+b^3-3xab$

(D) 0 for all x

Answer: (A)

2.5 Determine the correct value.

(A) 2^{12}

(B) 2

(C) 2^{14}

(D) 2^{26}

Answer: BONUS

Question 3

Let $f : \mathbb{R} \to \mathbb{R}$ be a C^2 function satisfying

$$f(0) = 0, \quad f'(0) = 1, \quad f''(x) = \int_0^x \frac{f'(t)}{1 + f(t)^2}\, dt \quad \text{for all } x \in \mathbb{R}.$$

3.1 Which of the following statements must be true?

(A) $f'(x) > 0$ for all $x \in \mathbb{R}$

(B) $f''(x) < 0$ for all $x > 0$

(C) f is bounded above on $(0, \infty)$

(D) f' has at least one zero on $(0, \infty)$

Answer: (A)

3.2 Which of the following statements is correct?

(A) f is concave on $(0, \infty)$

(B) f is strictly convex on $(0, \infty)$

(C) f'' changes sign infinitely many times on $(0, \infty)$

(D) $f''(x) = 0$ for all sufficiently large x

Answer: (B)

3.3 Define

$$H(x) = \int_0^x \frac{f'(t)}{1 + f(t)^2}\, dt.$$

Which of the following statements is correct?

(A) $H(x) \to \infty$ as $x \to \infty$

(B) $H(x)$ is bounded and $\lim_{x\to\infty} H(x) = \dfrac{\pi}{2}$

(C) $\lim_{x\to\infty} \dfrac{H(x)}{x} = 1$

(D) $H(x)$ oscillates infinitely often

Answer: (B)

3.4 Which statement correctly describes the growth of $f(x)$ as $x \to +\infty$?

(A) $f(x)$ is bounded for all $x > 0$

(B) There exists a constant $B > 0$ such that $f(x) \le Bx$ for all $x > 0$

(C) There exists a constant $B > 0$ such that $f(x) \ge Bx^2$ for all sufficiently large $x > 0$

(D) $\dfrac{f(x)}{x \log x}$ is unbounded

Answer: (C)

3.5 Define

$$\Phi(x) = f'(x)^2 - \log\bigl(1 + f(x)^2\bigr).$$

Which of the following statements is correct?

(A) $\Phi(x)$ is constant on $(0, \infty)$

(B) $\Phi(x)$ is strictly increasing on $(0, \infty)$

(C) $\Phi(x)$ is strictly decreasing on $(0, \infty)$

(D) $\Phi(x)$ oscillates infinitely often

Answer: (B)

Question 4

4.1 If the tracks have (effectively) infinite length, and $m = 2$, which of the following is an optimal value of A_1?

(A) 126

(B) 100

(C) 120

(D) 200

Answer: (A)

4.2 If the tracks have length 10 km, and $m = 2$, which of the following is an optimal value of A_1?

(A) 126

(B) 31

(C) 181

(D) 141

Answer: (B)

4.3 If the tracks have length 10 km, give the smallest value of m, to ensure no collision.

(A) 2

(B) 3

(C) 4

(D) 5

Answer: (A)

4.4 If the length of the track is 40 km, give the smallest value of m to ensure no collision.

(A) 2

(B) 3

(C) 4

(D) 5

Answer: (B)

4.5 Let the answer in the previous question be m_0. What is the minimum possible value of the maximum number of collisions?

(A) 1

(B) 5

(C) 10

(D) 15

Answer: (C)

Question 5

5.1 How many distinct numbers are there?

(A) 1

(B) 2

(C) 3

(D) 4

Answer: (A)

5.2 How many distinct numbers are there?

(A) 1

(B) 2

(C) 3

(D) 4

Answer: (B)

5.3 How many distinct numbers are there?

(A) 1

(B) 2

(C) 3

(D) 4

Answer: (C)

5.4 How many distinct numbers are there?

(A) 1

(B) 2

(C) 3

(D) 4

Answer: (C)

5.5 How many distinct numbers are there?

(A) 1

(B) 2

(C) 3

(D) 4

Answer: (C)

Question 6

6.1 Find the number of distinct real roots of $a_{100}(x)$ in $(-1, 1)$

(A) 100

(B) 95

(C) 1

(D) 98

Answer: (D)

6.2 Find the number of distinct real roots of $a_{100}(x) - a_{99}(x)$.

(A) 100

(B) 99

(C) 98

(D) 97

Answer: (A)

6.3 Find the number of real roots of $a_{50}(x) - a'_{50}(x)$

(A) 50

(B) 49

(C) 0

(D) 1

Answer: (A)

6.4 Find $\sum_{i=1}^{50} \operatorname{sgn}(\beta_i - \alpha_i)$

(A) 50

(B) 49

(C) 0

(D) -50

Answer: (B)

6.5 Find the number of critical points.

(A) $(n-1)(n-2)$

(B) $n(n-1)$

(C) 0

(D) $(n-1)^2$

Answer: (A)