SPARQ Junior 2026 Question Paper

Time: 120 minutes **Date: January 03, 2026**

Question 1

StreamLine, a video streaming platform with ten lakhs registered accounts, is currently evaluating a new pricing strategy to address high server costs. Under the current free, ad-supported model, the company earns Rs. 80 per hour in ad revenue against a streaming cost of Rs. 50 per hour, resulting in a net profit of Rs. 30 per hour watched. Aggregate usage data shows that the global average viewing time across all accounts is 2.0 hours per month, yet 50% of the total accounts show exactly zero activity. However, internal analysis reveals that this data is skewed by a "zero-inflated" user structure: 40% of the total accounts are "Ghost" users who have abandoned the platform and never watch content, while the remaining population consists of "Active" users who generally watch content but may occasionally register zero hours. The Chief Marketing Officer has proposed replacing the ad model with a flat Rs. 500 monthly subscription fee, featuring a "Data Saver Refund" where any user who watches exactly zero hours receives a full 100% refund. To evaluate the viability of this proposal, you must first use the provided global data to mathematically derive the average viewing time of the "Active" segment and the specific probability that an "Active" user watches zero hours. Using these derived figures, calculate the expected profit per account under the new model—accounting for the fact that zero-hour users generate no revenue and incur no costs—and determine if this strategy yields a higher expected profit than the current average of Rs. 60 per user.

1.1 Based on the usage data provided for **StreamLine**, calculate the average viewing time (in hours per month) of an *Active* user.

(A) 2.0

(B) 3.0

(C) 3.33

(D) 5.0

1.2 What is the probability that an *Active* user watches exactly 0 hours in a month?

(A) $\frac{1}{10}$

(B) $\frac{1}{6}$

(C) $\frac{1}{4}$

(D) $\frac{1}{3}$

1.3 Under the proposed subscription model with a full refund for zero usage, what is the expected profit per account (in rupees) for StreamLine?

(A) 60

(B) 100

(C) 150

(D) 50

1.4 Based on the expected profit calculations, which of the following statements is correct?

(A) The new subscription model is less profitable than the current ad-based model.

(B) The new subscription model yields the same expected profit as the current model.

(C) The new subscription model is more profitable than the current ad-based model.

(D) The profitability of the new model cannot be determined from the data.

1.5 Assuming that the behaviour of *Active* users remains unchanged, what is the *maximum percentage of Ghost users* the platform can have before the new subscription model becomes less profitable than the benchmark profit of Rs. 60 per account?

(A) 30%

(B) 36%

(C) 40%

(D) 50%

Question 2

Dr. Amory Roobi Dewitt discovers an ancient archival system known as the *Triadic Archive*, used to encode numerical information using 3×3 matrices. Each record in the archive is verified through a sequence of determinant-based consistency checks. The system is designed so that any violation in these checks immediately reveals corruption in the data.

The archive operates as follows:

2.1 The first layer of verification encodes three real parameters a, b, c (distinct) into the matrix

$$\Delta_1 = \begin{vmatrix} 1 & a & a^3 \\ 1 & b & b^3 \\ 1 & c & c^3 \end{vmatrix}.$$

The determinant Δ_1 acts as a cryptographic signature ensuring that the parameters are distinct and ordered.

Determine the value of Δ_1.

(A) $(a-b)(b-c)(c-a)$

(B) $(a-b)(b-c)(c-a)(a+b+c)$

(C) $(a-b)(b-c)(c-a)(a^2+b^2+c^2)$

(D) 0

2.2 To protect against uniform background noise, the archive allows the addition of a constant disturbance λ to quadratic data without affecting authenticity. This is tested using

$$\Delta_2(\lambda) = \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2+\lambda & b^2+\lambda & c^2+\lambda \end{vmatrix}.$$

If the determinant changes with λ, the record is deemed unstable.

Which of the following statements is true about $\Delta_2(\lambda)$?

(A) $\det \Delta_2(\lambda)$ depends on λ

(B) $\det \Delta_2(\lambda)$ is linear in λ

(C) $\det \Delta_2(\lambda)$ is independent of λ

(D) $\det \Delta_2(\lambda) = 0$

2.3 A decoding function $f : \mathbb{R} \to \mathbb{R}$ is used to map raw inputs to encrypted values. The archive mandates that for all real x, y, z,

$$\begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ f(x) & f(y) & f(z) \end{vmatrix} = 0.$$

This ensures that the encryption preserves structural dependence among entries.

Which of the following best describes the nature of the function $f(x)$?

(A) a constant function

(B) a linear function

(C) a quadratic function

(D) any polynomial

2.4 Certain records are stored in cyclic form to detect asymmetric tampering. Such a record is represented by

$$\Delta_3 = \begin{vmatrix} x & a & b \\ b & x & a \\ a & b & x \end{vmatrix}.$$

The determinant must factor into a symmetric and a non-symmetric component for the record to be valid.

Which of the following expressions correctly represents Δ_3?

(A) $(x + a + b)\big(x^2 + a^2 + b^2 - ab - bx - ax\big)$

(B) $(x - a - b)^3$

(C) $x^3 + a^3 + b^3 - 3xab$

(D) 0 for all x

2.5 At the deepest level, the archive stores a transformation matrix P with real entries and $\det(P) = 2$. To verify resistance against repeated inversion and scaling, the following quantity is computed:

$$\det\big(\operatorname{adj}\big(2 \cdot \operatorname{adj}(P^2)\big)\big).$$

A mismatch in this value signals irreversible corruption of the archive.

Determine the correct value.

(A) 2^{12}

(B) 2

(C) 2^{14}

(D) 2^{26}

Question 3

Let $f : \mathbb{R} \to \mathbb{R}$ be a C^2 function satisfying

$$f(0) = 0, \quad f'(0) = 1, \quad f''(x) = \int_0^x \frac{f'(t)}{1 + f(t)^2}\, dt \quad \text{for all } x \in \mathbb{R}.$$

3.1 Which of the following statements must be true?

(A) $f'(x) > 0$ for all $x \in \mathbb{R}$

(B) $f''(x) < 0$ for all $x > 0$

(C) f is bounded above on $(0, \infty)$

(D) f' has at least one zero on $(0, \infty)$

3.2 Which of the following statements is correct?

(A) f is concave on $(0, \infty)$

(B) f is strictly convex on $(0, \infty)$

(C) f'' changes sign infinitely many times on $(0, \infty)$

(D) $f''(x) = 0$ for all sufficiently large x

3.3 Define

$$H(x) = \int_0^x \frac{f'(t)}{1 + f(t)^2}\, dt.$$

Which of the following statements is correct?

(A) $H(x) \to \infty$ as $x \to \infty$

(B) $H(x)$ is bounded and $\lim_{x\to\infty} H(x) = \frac{\pi}{2}$

(C) $\lim_{x\to\infty} \frac{H(x)}{x} = 1$

(D) $H(x)$ oscillates infinitely often

3.4 Which statement correctly describes the growth of $f(x)$ as $x \to +\infty$?

(A) $f(x)$ is bounded for all $x > 0$

(B) There exists a constant $B > 0$ such that $f(x) \le Bx$ for all $x > 0$

(C) There exists a constant $B > 0$ such that $f(x) \ge Bx^2$ for all sufficiently large $x > 0$

(D) $\frac{f(x)}{x \log x}$ is unbounded

3.5 Define

$$\Phi(x) = f'(x)^2 - \log\bigl(1 + f(x)^2\bigr).$$

Which of the following statements is correct?

(A) $\Phi(x)$ is constant on $(0, \infty)$

(B) $\Phi(x)$ is strictly increasing on $(0, \infty)$

(C) $\Phi(x)$ is strictly decreasing on $(0, \infty)$

(D) $\Phi(x)$ oscillates infinitely often

Question 4

Suppose there are m race tracks, of same length, in a racing field. The field has 50 types of cars (sufficient number of each type to lend), n^{th} type of car has speed $5n$ m/s. Each driver comes to the field and is given a car randomly. If the speed of the car is between A_{k-1} and A_k m/s, $[A_0 = 0,\ A_m = 250]$, the car starts in k^{th} track. If a car overtakes another car with relative speed ≥ 50 m/s, the slower car gets damaged. A new driver comes after every 5 minutes in the field. Only 20 drivers come per day. A damaged car can't race further, and can't be damaged further.

4.1 If the tracks have (effectively) infinite length, and $m = 2$, which of the following is an optimal value of A_1?

(A) 126

(B) 100

(C) 120

(D) 200

4.2 If the tracks have length 10 km, and $m = 2$, which of the following is an optimal value of A_1?

(A) 126

(B) 31

(C) 181

(D) 141

4.3 If the tracks have length 10 km, give the smallest value of m, to ensure no collision.

(A) 2

(B) 3

(C) 4

(D) 5

4.4 If the length of the track is 40 km, give the smallest value of m to ensure no collision.

(A) 2

(B) 3

(C) 4

(D) 5

4.5 Let the answer in the previous question be m_0. Consider now that we have $m_0 - 1$ many tracks. What is the minimum possible value (depending upon A_i's) of the maximum number of collisions?

(A) 1

(B) 5

(C) 10

(D) 15

Question 5

Rohan opens a statistics book for the first time, and gets overwhelmed by tons of formulae written there. But gradually realises that many of the formulae are essentially the same. Help him to find the number of different formulae. Suppose $x_1, x_2, \ldots, x_n, y_1, \ldots, y_n$ are distinct real numbers.

5.1 How many distinct numbers are there in

$$\left\{\frac{1}{n}\sum_{i=1}^{n}(x_i-\bar{x})^2,\ \frac{1}{n}\sum_{i=1}^{n}x_i^2-\bar{x}^2,\ \frac{1}{2n^2}\sum_{i\neq j}(x_i-x_j)^2,\ \frac{1}{n}\sum_{i=2}^{n}\frac{i}{i-1}(x_i-\bar{x})^2\right\}?$$

(A) 1

(B) 2

(C) 3

(D) 4

5.2 How many distinct numbers are there in

$$\left\{\sum_{i=1}^{n} ix_i,\ \sum_{i=1}^{n}\left(\sum_{j=i}^{n}x_j\right),\ \sum_{i=1}^{n}\left(\sum_{j=1}^{i}x_i\right),\ \sum_{i=1}^{n}(n-i)x_i\right\}?$$

(A) 1

(B) 2

(C) 3

(D) 4

5.3 How many distinct numbers are there in

$$\left\{\sum_{i=1}^{n}(\frac{\sum_{j=1}^{i}x_j}{i}),\ \frac{\sum_{i=1}^{n}x_i}{n},\ \arg\min_{y}\frac{\sum_{i=1}^{n}(x_i-y)^2}{n},\ \arg\max_{y}\frac{\sum_{i=1}^{n}(x_i-y)^{-2}}{n}\right\}?$$

(A) 1

(B) 2

(C) 3

(D) 4

5.4 How many distinct numbers are there in

$$\left\{\arg\min_{\alpha}\sum_{i=1}^{n}(y_i-\alpha x_i)^2,\ \arg\min_{\alpha}\sum_{i=1}^{n}(y_i-\alpha x_i)^4,\ \frac{\sum_{i=1}^{n}x_iy_i}{\sum_{i=1}^{n}x_i},\ \frac{\sum_{i=1}^{n}x_i^3y_i^3}{\sum_{i=1}^{n}x_i}\right\}?$$

(A) 1

(B) 2

(C) 3

(D) 4

5.5 How many distinct numbers are there in

$$\left\{ \operatorname{med}(x_1, \ldots, x_n),\ \arg\min_c \sum_{i=1}^{n} |x_i - c|,\ \sqrt{\arg\min_c \sum_{i=1}^{n} |x_i^2 - c|},\ \sin^{-1}\left(\arg\min_c \sum_{i=1}^{n} |\sin(x_i) - c| \right) \right\}?$$

(A) 1

(B) 2

(C) 3

(D) 4

Question 6

Define a sequence of real polynomials $(a_n)_{n\geq 1}$ by

$$a_1(x) = x, \qquad a_{n+1}(x) = (x^2-1)\, a_n'(x) \quad (n \geq 1).$$

6.1 Find the number of distinct real roots of $a_{100}(x)$ in the interval $(-1, 1)$

(A) 100

(B) 95

(C) 1

(D) 98

6.2 Find the number of distinct real roots of $a_{100}(x) - a_{99}(x)$.

(A) 100

(B) 99

(C) 98

(D) 97

6.3 Find the number of real roots of $a_{50}(x) - a_{50}'(x)$

(A) 50

(B) 49

(C) 0

(D) 1

6.4 Let the roots of a_{100} be

$$\alpha_1 < \alpha_2 < \cdots < \alpha_{n-1} < \alpha_n,$$

and the roots of a_{99} be

$$\beta_1 < \beta_2 < \cdots < \beta_{n-1}.$$

Then find $\sum_{i=1}^{50} sgn(\beta_i - \alpha_i)$

(A) 50

(B) 49

(C) 0

(D) -50

6.5 Define a function $f : \mathbb{R}^2 \to \mathbb{R}$ by

$$f(x, y) = a_n(x)\, a_{n-1}(y).$$

Find the number of points where the ordered pair

$$\left(\frac{\partial f}{\partial x}(x,y), \frac{\partial f}{\partial y}(x,y)\right)$$

vanishes in the open square $(-1,1)\times(-1,1)$, and that each such point lies strictly inside a rectangle of the form

$$(\alpha_i, \alpha_{i+1}) \times (\beta_j, \beta_{j+1}), \qquad 1 \le i \le n-1,\ 1 \le j \le n-2.$$

(A) $(n-1)(n-2)$

(B) $n(n-1)$

(C) 0

(D) $(n-1)^2$

All the best!!