Problem Name:
Problem Code:

MCQ-1

You are in line at an ice cream shop when the manager announces that she will give a free sundae to the first person in line whose birthday is the same as someone who has already bought an ice cream. Assuming that you do not know anyone else's birthday and that all birthdays are uniformly distributed across the 365 days in a normal year, what position in line will you choose to maximize your probability of receiving the free sundae?

A) 19

B) 20

C) 21

D) None of these

Problem Name:
Problem Code:

MCQ-2

Consider a linear regression model with n observations and p covariates. We compare two different regularization approaches with a tuning parameter $\lambda > 0$:

Model 1 (Bridge Regression): A regression fit with an L_q penalty where $1 < q < 2$. Let the fitted parameter vector be $\hat{\beta}_1$.

Model 2 (Elastic Net): An Elastic Net fit with a mixing parameter $\alpha = 2 - q$. The penalty is defined as $\lambda \left[\alpha\|\beta\|_1 + (1-\alpha)\|\beta\|_2^2\right]$.

Let the fitted parameter vector be $\hat{\beta}_2$.Let $L(\lambda) = \|\hat{\beta}_1\|_0 - \|\hat{\beta}_2\|_0$, where $\|\cdot\|_0$ denotes the L_0 norm (the number of non-zero components in the vector).Which of the following statements are correct? (More than 1 correct type)

A) $L(\lambda)$ is a non-decreasing function of λ.

B) $\|\hat{\beta}_1 - \hat{\beta}_2\|_2 \to 0$ as $\lambda \to \infty$.

C) $\|\hat{\beta}_1 - \hat{\beta}_2\|_0 \to 0$ as $\lambda \to \infty$.

D) The maximum possible value of $L(\lambda)$ is $p - 1$.

Problem Name:
Problem Code:

MCQ-3

3 points are randomly chosen on the circumference of a circle of radius r, what is the probability that the triangle formed by joining them is acute?

A) $\frac{1}{4}$

B) $\frac{3}{4}$

C) $\frac{1}{2}$

D) Will depend on the value of r

Problem Name:
Problem Code:

MCQ-4

You are playing a one-player game with two opaque boxes. At each turn, you can choose either Place or Take.

-Place puts 1 from a third party into one of the two boxes, chosen uniformly at random.
-Take empties one of the two boxes, chosen uniformly at random, and the money in that box becomes yours.

The game consists of 100 turns, and at each turn you must choose either Place or Take. Assuming optimal play, if E is the expected payoff of this game, then report $\lfloor E \rfloor$.

Note that you do not know how much money you have taken until the end of the game.

A) 49

B) 62

C) 73

D) 92

Problem Name:
Problem Code:

MCQ-5

Let $X_1, X_2, \ldots, X_n$ be i.i.d. observations from a $N(\mu, \sigma^2)$ distribution, where σ^2 is known. To test the hypothesis

$H_0 : \mu = \mu_0$ against $H_1 : \mu \neq \mu_0$

we use the test statistic: $T(X) = \frac{\bar{X}-\mu_0}{\sigma/\sqrt{n}}$

For a given $\alpha \in (0,1)$ and a weight parameter $q \in (0,1)$, the decision rule is to reject H_0 if: $T(X) > \Phi^{-1}(1-\alpha q) \quad \text{or} \quad T(X) < \Phi^{-1}(\alpha(1-q))$where $\Phi(\cdot)$ denotes the Cumulative Distribution Function (CDF) of the standard normal distribution. Let $f(q)$ be the p-value of the observed data calculated as a function of $q \in (0,1)$.Which of the following statements are correct? (More than 1 correct type)

A) $f(q)$ is a piecewise linear and non-differentiable function in $(0,1)$.

B) $f(q)$ is differentiable everywhere in the interval $(0,1)$.

C) $f(q)$ attains a unique maximum in the interval $(0,1)$.

D) If q is a random variable such that $q \sim U(0,1)$, then: $E[f(q)] = -\Phi(T)\ln(\Phi(T)) - (1-\Phi(T))\ln(1-\Phi(T))$

Problem Name:

Problem Code:

MCQ-6

Let $V = \mathbb{C}^d$ be a d-dimensional inner product space with the standard inner product. Two orthonormal bases $\mathcal{B} = \{e_1, \ldots, e_d\}$ and $\mathcal{C} = \{f_1, \ldots, f_d\}$ are said to be Mutually Unbiased Bases (MUB) if:$|\langle e_i, f_j \rangle|^2 = \frac{1}{d} \quad \text{for all } i, j.$

Fix $i \in \{1, \ldots, d\}$ and write:

$$e_i = \sum_{j=1}^{d} a_j f_j, \quad \text{where } a_j = \langle e_i, f_j \rangle.$$

Assume that $\mathcal{B}$ and $\mathcal{C}$ are mutually unbiased. Which of the following statements is necessarily true?

A) $\sum_{j=1}^{d} a_j = 0$

B) $\sum_{j=1}^{d} |a_j|^4 = \frac{1}{d}$

C) $\sum_{j=1}^{d} |a_j|^4 = \frac{1}{d^2}$

D) $\sum_{j=1}^{d} a_j^2 = 1$

Problem Name:

Problem Code:

MCQ-7

Let $V = \mathbb{C}^d$ be a d-dimensional inner product space with the standard inner product. Two orthonormal bases $\mathcal{B} = \{e_1, \ldots, e_d\}$ and $\mathcal{C} = \{f_1, \ldots, f_d\}$ are said to be Mutually Unbiased Bases (MUB) if: $|\langle e_i, f_j \rangle|^2 = \frac{1}{d} \quad \text{for all } i, j.$

For each i, j, define the rank-one projection matrices: $P_i = e_i e_i^*, \quad Q_j = f_j f_j^*$ where vectors are viewed as column vectors and $(\cdot)^*$ denotes the conjugate transpose. Which of the following conditions is equivalent to the statement that $\mathcal{B}$ and $\mathcal{C}$ are mutually unbiased?

A) $\mathrm{Tr}(P_i Q_j) = 0$ for all $i \neq j$

B) $\mathrm{Tr}(P_i Q_j) = \frac{1}{d}$ for all i, j

C) $P_i Q_j = Q_j P_i$ for all i, j

D) $\sum_{i=1}^{d} P_i Q_i = I$

Problem Name:

Problem Code:

MCQ-8

Let $V = \mathbb{C}^d$ be a d-dimensional inner product space with the standard inner product. Two orthonormal bases $\mathcal{B} = \{e_1, \ldots, e_d\}$ and $\mathcal{C} = \{f_1, \ldots, f_d\}$ are said to be Mutually Unbiased Bases (MUB) if:$|\langle e_i, f_j \rangle|^2 = \frac{1}{d} \quad \text{for all } i, j.$

Let $\mathcal{B}$ and $\mathcal{C}$ be orthonormal bases of $\mathbb{C}^d$ and define:

$M(\mathcal{B}, \mathcal{C}) := \max_{i,j} |\langle e_i, f_j \rangle|^2$

Which of the following statements is correct?

A) $M(\mathcal{B}, \mathcal{C}) \geq \frac{1}{d}$, with equality if and only if $\mathcal{B}$ and $\mathcal{C}$ are mutually unbiased.

B) $M(\mathcal{B}, \mathcal{C}) \leq \frac{1}{d}$ for all orthonormal bases.

C) $M(\mathcal{B}, \mathcal{C}) = 1$ if and only if the bases are mutually unbiased.

D) $M(\mathcal{B}, \mathcal{C})$ is independent of d.

Problem Name:
Problem Code:

MCQ-9

There are 2026 distinct balls kept in a single line. In a given move 2 balls are randomly chosen and swapped. What is the probability that after 2026 moves, the balls will be one swap away from their original orientation?

A) $\frac{1}{2}$

B) $\frac{571}{2026}$

C) $\frac{1}{2026}$

D) 0

Problem Name:
Problem Code:

MCQ-10

A proprietary trading desk operates a binary daily trading strategy on a stock index. Each trading day results in one of two possible outcomes: a profitable day, denoted by W, or a losing day, denoted by L.

Over a historical backtest covering 100 trading days, the strategy records a total of 60 profitable days and 40 losing days. It is assumed that the sequence of wins and losses is generated in a random order, and that there is no serial dependence in the outcomes, in particular, the strategy exhibits neither momentum nor mean-reversion effects across consecutive days.

A run is defined as a maximal sequence of identical outcomes occurring on consecutive trading days. For example, a sequence of three profitable days in a row (WWW) constitutes a single run, while an alternating sequence such as WLW consists of three runs.

Under these assumptions, what is the expected number of runs in the 100-day trading record?

A) 51

B) 49

C) 50

D) 77

Problem Name:
Problem Code:

MCQ-11

In Principal Component Analysis (PCA), we seek a direction vector $u \in \mathbb{R}^p$ that maximizes the variance of the projected data. If Σ is the sample covariance matrix, this is formulated as:

$$\text{Maximize } u^T \Sigma u \quad \text{subject to } u^T u = 1$$

Using the method of Lagrange Multipliers, let $\mathcal{L}(u, \lambda) = u^T \Sigma u - \lambda(u^T u - 1)$. Which of the following statements are correct?

A) The stationary points of $\mathcal{L}$ satisfy the equation $\Sigma u = \lambda u$.

B) The maximum variance is equal to the largest eigenvalue of Σ.

C) If Σ is a rank-deficient matrix, there is no solution to this optimization problem.

D) The Lagrange multiplier λ represents the variance of the data projected onto the direction u

Problem Name:
Problem Code:

MCQ-12

In training a neural network or fitting a generalized linear model, we often minimize a loss function $L(\theta)$ where $\theta \in \mathbb{R}^p$. Let $H(\theta) = \nabla^2 L(\theta)$ be the Hessian matrix of the loss function at a critical point θ^* (where $\nabla L(\theta^*) = 0$).Suppose $L(\theta)$ is twice continuously differentiable. Which of the following statements are correct?

A) If $H(\theta^*)$ has at least one positive and one negative eigenvalue, θ^* is a saddle point.

B) If $H(\theta^*)$ is positive definite, the Newton-Raphson update step $\Delta\theta = -H(\theta^*)^{-1}\nabla L(\theta)$ points in a descent direction.

C) For a ridge regression objective $L(\beta) = \|y - X\beta\|_2^2 + \lambda\|\beta\|_2^2$, the Hessian $H(\beta)$ is independent of β.

D) If $L(\theta)$ is a strictly convex function, then $H(\theta)$ must be singular for all θ.

Problem Name:
Problem Code:

MCQ-13

What is the least number of colours required to color the integers $1, 2, ..., 2^n$ such that in any set of consecutive integers, there is a colour that occurs exactly once?

That is, the colouring has to satisfy the condition that for any integers i, j such that $1 \leq i \leq j \leq 2^n$, there is a colour that is given to exactly one integer in the set $i, i+1, ..., j-1, j$.

A) $n+1$

B) n

C) $n-1$

D) $\frac{n}{2}$

Problem Name:
Problem Code:

MCQ-14

Let X and Y be two independent standard normal random variables, $X, Y \sim N(0,1)$. We apply the following transformation to represent them in polar coordinates (R, Θ):

$X = R\cos\Theta \ Y = R\sin\Theta$

where $R \in [0, \infty)$ and $\Theta \in [0, 2\pi)$. Let J be the Jacobian matrix of the transformation from (R, Θ) to (X, Y). Which of the following is true regarding the absolute value of the determinant of the Jacobian, $|\det(J)|$, and the resulting joint density?

A) $|\det(J)| = R^2$

B) $|\det(J)| = R$

C) The joint density of (R, Θ) is independent of Θ, implying Θ is uniformly distributed on $[0, 2\pi)$.

D) The transformation is non-invertible at the origin, so the Jacobian is undefined for all $R > 0$.

Problem Name:

Problem Code:

MCQ-15

The sequence given by $x_0 = a$, $x_1 = b$, and

$x_{n+1} = \frac{1}{2}\left(x_{n-1} + \frac{1}{x_n}\right)$ is periodic.

The value of ab is

A) -1

B) $\frac{1}{2}$

C) Cannot be determined using the given information.

D) None of these.

Problem Name:

Problem Code:

MCQ-16

Consider the following game played on an infinite grid with square cells (like an infinite chessboard in which all squares are white). First, some n cells that are chosen arbitrarily are coloured black. The game now begins. In each round, some cells coloured black change to white, some cells coloured white change to black and other cells remain the same according to the following rule: the colour of a cell is toggled if and only if its current colour is different from the colour of the cell to its right and the colour of the cell above it. After how many rounds will every cell necessarily become white?

A) $n+1$

B) n

C) $n-1$

D) $\frac{n}{2}$

Problem Name:
Problem Code:

MCQ-17

Suppose we are given a training dataset $\{(x_i, y_i)\}_{i=1}^n$ where $x_i \in \mathbb{R}^p$ and labels $y_i \in \{+1, -1\}$. We fit two binary classification models by minimizing the following objective functions:

Model 1 (Logistic Regression): $\min_\beta \sum_{i=1}^n \ln(1 + \exp(-y_i x_i^T \beta)) + \frac{\lambda}{2}\|\beta\|_2^2$

Model 2 (Soft-Margin SVM): $\min_\beta \sum_{i=1}^n \max(0, 1 - y_i x_i^T \beta) + \frac{\lambda}{2}\|\beta\|_2^2$

We initially fit both models on the n points. We then add 1,000 new observations to the training set. It is observed that for every new point (x_{new}, y_{new}), the condition $y_{new} x_{new}^T \hat{\beta} > 1$ holds for both the current optimal Logistic Regression estimate and the current SVM estimate.Which of the following statements are correct?

A) For a small λ, adding these 1,000 points changes the optimal β for the SVM model, but not for the Logistic Regression model.

B) For a small λ, adding these 1,000 points changes the optimal β for the Logistic Regression model, but not for the SVM model.

C) The magnitude of the shift in the fitted parameter vector β (for either model) decreases as the regularization parameter λ is increased.

D) The magnitude of the shift in the fitted parameter vector β (for either model) increases as the regularization parameter λ is increased.

Problem Name:
Problem Code:

Hunter Spider

Arachnia, a hungry spider is looking for a prey in a narrow stick, while Fling, a fly nearby has seen the predator approaching. Arachnia has still not seen any fly, so it is jumping randomly. The stick is narrow enough that the spider can only move back and forth along one line only, assume it to be the X-axis, with the current position of the spider as origin.

Also assume that the spider jumps along positive X direction with probability p and negative X direction with probability $1 - p$, and the magnitude of the jump is distributed as Uniform(0,l), where $0 < p < 1$, $l > 0$ are unknown parameters, all the jumps are independent and direction and magnitude of each jump are independent. The fly looks at the position of the spider for 10 consecutive jumps, and decides to go as far as possible from the position of the spider after the next jump. The fly tries to get a reasonable estimate as fast as possible - Calculate the MLEs of p, l assume them to be the true values of the parameters and calculate the expectation of the position after the next jump as an estimate.

Your answer will be considered correct if the absolute or relative error does not exceed 10^{-6}.

Input Format

The input consists of 10 integers, each given on a separate line, representing the positions of the spider after 10 consecutive jumps.

Output Format

The predicted position after next jump.

Sample 1:

Input	Output
3 4 1 -2 8 4 6 5 4 7	7.0

Sample 2:

Input	Output
1.2 2.1 3.8 2.6 4.89 5.62 6.99 7.81 9.21 8.98	9.667000000000002

Problem Name:
Problem Code:

Red Circle, Blue Circle

There are $3N$ positive integers (not necessarily distinct) written on a board. Alice and Bob play a game with N rounds. In each round, Alice circles a number with red colour while Bob circles a number with blue colour and erases some number that has not been coloured earlier. After N rounds, there are N numbers in red circles, and N numbers in blue circles. Alice wins if the sum of the numbers in red circles is not equal to the sum of the numbers in blue circles, while Bob wins if the two sums are equal.

Input Format

The first line contains an integer N.

The second line contains $3N$ integers (separated by whitespace character)

Output Format

Assuming both Alice and Bob play optimally, output whether Alice has a winning strategy or not as Y/N .

Constraints

$$0 \leq N \leq 10^6$$

Sample 1:

Input	Output
4 1 1 1 1 1 1 1 1 1 1 1 1	N

Sample 2:

Input	Output
2 1 2 4 8 16 32	Y

Problem Name:
Problem Code:

Scam Alert

The Scam Intelligence Agency (SIA) of a city has information about an ongoing scam in the neighboring cities. A foreigner comes to the city and selects a person and asks him for 100 Rs and tells that person to ask 100 Rs from two of his friends and to tell them to ask two of their friends and so on. If every body manages to get the money from two friends then everybody is at profit. SIA immediately identifies this as a scam.

Given any person from the population, SIA knows which two people they will ask for money. SIA also knows that the people in the city are susceptible to the scam only once. Once they give money, they are no longer going to give or take money anymore. For example, in the picture below we depict a town with five people.

An arrow from A to B indicates that A would ask B for the money. In this example, B can lose money. We can check that with the following scenario.

1. Someone from outside the town asks A for money.
2. A asks B for money.
3. A asks C for money.
4. C asks D for money.
5. B asks C for money.
6. B asks D for money.

Observe that when B asks C and D for money, they will not give it to B since they have already given money to someone else.

Your answer will be considered correct if the absolute or relative error does not exceed 10^{-6}.

Input Format

The first line contains an integer N (3 ≤ N ≤ 1000) indicating the number of people in the city. Each person is identified by a distinct integer from 1 to N. For i = 1, 2, . . . , N, the i-th of the next N lines contains two integers Xi and Yi (1 ≤ Xi, Yi ≤ N, Xi, Yi 6= i and Xi 6= Yi), representing that person i would ask for money to person Xi and person Yi

Output Format

Output a single line with a string of length N such that its i-th character is the uppercase letter “Y” if person i can lose money, and the uppercase letter “N” otherwise

Constraints

- $3 \leq n$

Sample 1:

Input	Output
5 2 3 3 4 4 5 5 1 1 2	YYYYY

Sample 2:

Input	Output
4 2 3 3 4 2 4 2 3	NYYY

Sample 3:

Input	Output
4 2 3 3 1 1 2 2 3	YYYN